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## Area-Based COI-Referred Transient Stability Index for Large-Scale Power System



**Abstract—** This paper presents a new transient stability index called the Area-based COI-referred Transient Stability Index for a large electrical power system. A large power system is divided into smaller areas depending on the coherency of the system due disturbances before the index is applied on the system. The proposed index is defined by associating with each area of the power system an equivalent inertia representing the total inertia of the generation located in that area. Assuming that each area is coherent, it is possible to assimilate its behavior to that of a single large machine with same inertia and generation. It also offers a direct means of deriving the centre of inertia (COI). The COI provides very useful information for tracking the stability of interconnected areas. So, instead of assessing all the generators' rotor angles in the system, it is sufficient to assess only the area-based COI referred transient stability index which represents a single large machine rotor angle of each area. Simulations on the large practical power system show the effectiveness of the proposed index.

**Keywords –** Transient stability index, transient stability assessment, time domain simulations.

### I. INTRODUCTION

Power system stability denotes the ability of an electric power system, for a given initial operating condition, to regain a state of operating equilibrium after being subjected to a physical disturbance, with most system variables bounded so that practically the entire system remains intact [1]. Power system stability studies have been divided into rotor angle, voltage and frequency stabilities. Rotor angle stability is divided into two smaller categories: small signal transient stabilities [2]. The focus of this paper is on the matter of transient stability. Transient stability assessment (TSA) is part of dynamic security assessment of power systems which involves the evaluation of the ability of a power system to maintain synchronism under severe but credible contingencies. One of the most important roles of TSA is to formulate a transient stability index (TSI) for the contingencies, which is used to assess the stability of power systems and to rank the severity of the contingencies. Methods normally employed to formulate TSI are by using time domain simulations, energy function, hybrid and artificial intelligence methods. Time domain simulation method is implemented by solving the state space differential equations of power networks and then determines transient stability. Energy function method determines transient stability by calculating the stability margin of power systems. The hybrid method combined time domain simulations and energy function methods in calculating the TSI.

Hybrid method is used in reference [3] to develop a new approach to TSA called Marginally Unstable Injection (MUI) to develop a more accurate TSI. Reference [4] used a hybrid approach to identify critical machine and to build TSI

which is able to classify whether the power system is stable or unstable. Another TSI proposed in reference [5] which is also called the hybrid transient energy margin is developed by incorporating time domain simulations with the transient energy functions. The TSI developed in reference [5] is based on the implicit expression of the stability region of power system related to the controlling unstable equilibrium point. This index is used to calculate critical clearing time and contingencies ranking of the power system.

This paper concerns the TSA of a large power system. A new transient stability index called the Area-based COI-referred Transient Stability Index for a large electrical power system is proposed. Before applying the proposed index, the power system is divided into smaller areas depending on the coherency of the system due to disturbances. The proposed index is defined by associating with each area of the power system an equivalent inertia representing the total inertia of the generation located in that area. Assuming that each area is coherent, it is possible to assimilate its behavior to that of a single large machine with same inertia and generation. So, instead of assessing all generators' rotor angles in the system, the index reduced to only assessing the single large machine rotor angle's representing each area. This work is similar with the work in reference [6], the difference in this paper is the rotor angles data for each generator in the system are measured and recorded through time domain simulations whereas reference [6] uses recorded phase angle data from phasor measurement units located strategically in the power system. Simulations on the large practical power system will illustrate the effectiveness of the proposed index. The simulations tools used for this work are PSS<sup>TM</sup>E and MATLAB programs.

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## II. FORMULATION OF THE TRANSIENT STABILITY INDEX

Before deriving the proposed TSI, the machine equations used to evaluate the dynamic behavior of a power system for transient stability simulation is first considered. The differential equations to be solved in power system stability analysis using the time domain simulation method are the nonlinear ordinary equations with known initial values. Using the classical model of machines, the dynamic behavior of an  $n$ -generator power system can be described by the following equations:

$$M_i \frac{d^2 \delta_i}{dt^2} = P_{mi} - P_{ei} \quad (1)$$

However,

$$\dot{\delta}_i = \omega_i \quad (2)$$

By substituting (2) in (1), equation (1) becomes

$$M_i \dot{\omega}_i = P_{mi} - P_{ei} \quad (3)$$

where  $\delta_i$  - rotor angle of machine  $i$ ;  $\omega_i$  - rotor speed of machine  $i$ ;  $P_{mi}$  - mechanical power of machine  $i$ ;  $P_{ei}$  - electrical power of machine  $i$  and  $M_i$  - moment of inertia of machine  $i$ .

Equation (3) is then solved by using a time domain simulation program through step-by-step integration so as to produce time response of all the state variables. The rotor angles calculated from equation (3) are then used in the computation of the proposed TSI.

The TSI is associated with each area of a power grid and is based on an equivalent inertia representing the total inertia of the generators located in that area. Assuming that all the generators in an area is coherent following a disturbance, it is reasonable to assume an equivalent single large machine representing all the generators in that area. This can be made possible by deriving the centre of inertia (COI), which is very useful information for tracking the stability of interconnected areas. Before defining the COI quantities, the area equivalent rotor angle concept needs to be defined.

For a particular area, with  $N$  number of generators, that particular area equivalent rotor angle is the average rotor angle through all  $N$  measurements which is given by,

$$\bar{\delta}_j = \frac{1}{N} \sum_{i=1}^N \delta_i \quad (4)$$

where  $\bar{\delta}_j$  is the area equivalent rotor angle and  $\delta_i$  is the individual rotor angle in the particular area. Then assuming a total number of  $r$  areas in a power system, the COI of the system can be defined as,

$$\bar{\delta}_{COI}(t) = \frac{1}{M_T} \sum_{j=1}^r M_j \bar{\delta}_j, \quad M_T = \sum_{j=1}^r M_j \quad (5)$$

where  $M_T$  is the total inertia of the system and  $M_j$  is the  $j$ -th area inertia.

Therefore, the area equivalent rotor angle is expressed in the COI frame and is given by:

$$\delta_j^{COI} = \bar{\delta}_j - \bar{\delta}_{COI} \quad (6)$$

The area-based COI-referred rotor angle as given in (6) is then used as a TSI in which the index can be illustrated by plotting it against time. From the plot, it can be observed that if the rotor angle of any area ( $\delta_i^{COI}$ ) goes out of step after a fault is cleared then the area is said to be unstable whereas if it remains in equilibrium then the area is said to be stable.

## III. TEST SYSTEM DESCRIPTION

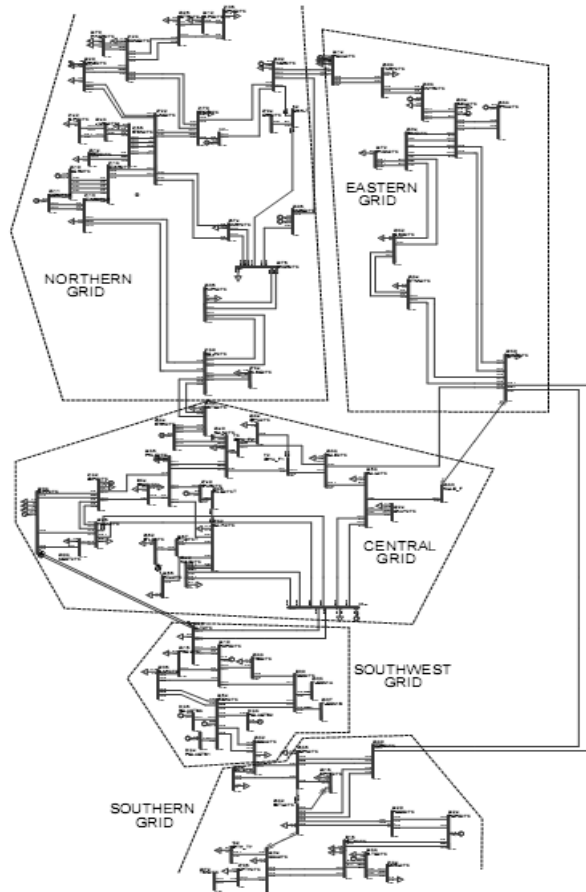


Fig. 1: The large practical power system with five electrically coherent areas

The proposed index has been examined on a large practical power system. Fig. 1 shows the large practical power system consists of 23 synchronous generators, 87 buses and 177 transmission lines. The system is divided into five areas; which are Northern Grid (6), Central Grid (6), Southwest Grid (2), Southern Grid (5) and Eastern Grid (4). The number in the brackets represents the number of generators in the respective grids. The areas are divided according to generators coherency due to disturbances.

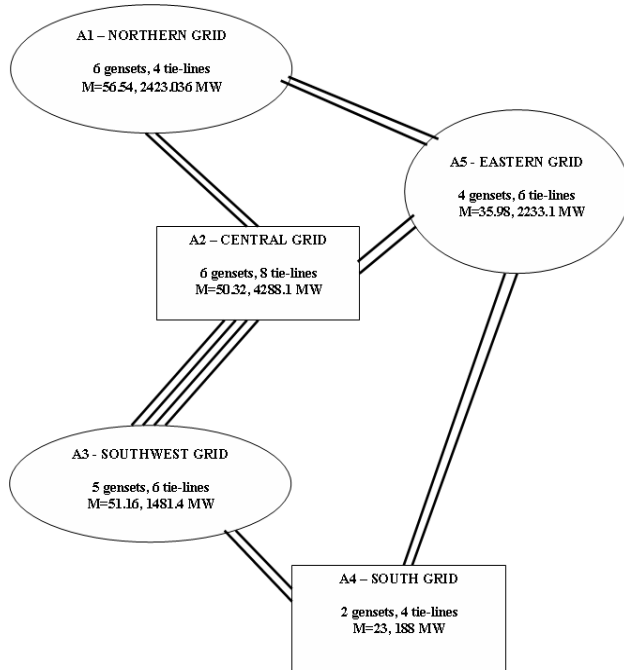


Fig. 2 Simplified version of the practical power system

Fig. 2 represents a simplified version of the test power system so as to highlight the interconnections between the five areas and number of generators, generation (MW) and inertia constant for respective areas.

The system is developed in PSS<sup>TM</sup>E. In the simulation model, the generator is modeled by 6<sup>th</sup> order differential equations and the loads are constant impedance. Model of excitation system is IEEE-type 1 with turbine governors type-1.

#### IV. TRANSIENT STABILITY SIMULATION ON THE TEST SYSTEM

Time domain simulation method is used in this work to assess the transient stability of the large practical power system because it is the most reliable, mature and accurate method compared to other method. The differential equations to be solved in transient stability analysis are nonlinear ordinary equations with known initial values. For this purpose the PSS<sup>TM</sup>E software is used.

In this work, the dynamic performance of the system during disturbances is based on observation of the rotor angle of generators in their respective areas via a time domain simulation method. About 114 three-phase faults are applied to the system. The three-phase faults are created at various locations in the system at any one time. In this aspect, the power system goes through prefault, fault-on and postfault stages [7]. When a three-phase fault occur at any line in the system, a breaker will operate and the respective line will be disconnected at the fault clearing time (FCT) which is set at 100 ms [8]. The key question in transient stability assessment is whether the transient swings are finally convergent or divergent. According to reference [9], if the relative rotor angles remain stable after a fault is cleared, it implies that the power system is stable but if the relative angles go out of step after a fault is cleared, it means that the system is unstable. The time step,  $\Delta t$ , for the time domain simulations is set at 0.02 seconds. The time frame of

interest in transient stability studies is usually limited to 3 to 5 seconds following the disturbance; it may be extended to 10 seconds for very large systems with dominant inter-area swings [2].

All the rotor angles data collected from all the contingencies are then applied to the proposed TSI. The TSI are then plotted to illustrate the severity of the contingencies on the power system.

#### V. TEST RESULTS

The time taken to run the simulations is set at 20 seconds since it is a large system. Collected data from PSS<sup>TM</sup>E are then applied to the TSI to illustrate the effectiveness of the proposed index. The proposed TSI is plotted against time using MATLAB program. The area-based COI-referred rotor angles are then compared with the rotor angles plotted for all the generators in the five areas of the system. Average voltage for each area is also plotted to illustrate the effect of proposed TSI on the voltage. For illustration purposes, only a few selected contingencies (5 cases) are shown to prove the accuracy of the proposed index.

*A. Case 1* - A three-phase fault is first created at bus 73 in the Central Grid. After the fault is cleared at 100 ms, the transmission line connecting bus 73 and bus 2338 in the same area is disconnected. Fig. 3 shows the six plots consisting of individual rotor angle plots for each machine in their respected areas (Fig. 3(a) – (e)) and the area-based COI-referred rotor angles (Fig. 3(f)). From the plots, it shows that the fault does not affect the system stability since the individual rotor angle plots of all machines in their respective areas do not violate the stability limit of the power system. The COI-referred rotor angles plots are also consistent with the individual rotor angles plots and they remain in synchronism after the fault has occurred. Fig. 4 shows the proposed TSI and average area voltages response signals due to the disturbances. It shows that the fault is weak on the system since the COI-referred rotor angles remain in synchronism after the fault had occurred. This illustrates a transiently stable case.

*B. Case 2* - A three-phase fault is created at bus 132 in the Southern Grid. After the fault is cleared at 100 ms, the transmission line connecting bus 132 and bus 2494 in the same area is disconnected. Fig. 5 shows the six plots consisting of individual rotor angle plots for each machine in their respected areas (Fig. 5(a) – (e)) and the proposed TSI (Fig. 5(f)). It can be seen from Fig. 5(a), (b), (c) and (e) that the angles increase uniformly except for the rotor angle in the Southern Grid (Fig. 5(d)). The increase for both rotor angles is not uniform and one of the generators is leaving the other generator exponentially which in turn will violate the stability limit of the area. The COI-referred rotor angles plot (Fig. 5(f)) is also consistent with the individual rotor angle plots. All the COI-referred rotor angles except for that in the Southern Grid remain intact after the fault has occurred. The Southern Grid stability status is lost since the COI-referred rotor angle goes out of step after the disturbance. This case is considered as not a severe contingency case since the power system is still intact after the disturbance. However, it should provide a warning to the operators of an imminent

transiently unstable case. Fig. 6 shows the proposed TSI and average area voltages response signals due to the fault. The average Area 4 voltage is reduce to about 0.95 p.u., usually the transmission voltage need to be maintained at 1.0 p.u. for efficient operation.

C. Case 3 - A three-phase fault is created at bus 2118 in the Northern Grid. After the fault is cleared at 100 ms, the transmission line connecting bus 2118 and bus 2119 in the same area is disconnected. Fig. 7 shows the six plots consisting of individual rotor angle plots for each machine in their respected areas (Fig. 7(a) – (e)) and the proposed TSI (Fig. 7(f)) when subjected to the fault. It can be seen from Fig. 7(a) – (e) that all rotor angles in their respective areas increases exponentially and not uniform. Due to these faults, the rotor angles increase very quickly and the plots of the angles imply that the system becomes unstable. The COI-referred rotor angles plot shown in Fig. 7(f) is also consistent with the individual rotor angle plots. All the areas COI-referred rotor angles go out of step after the fault has occurred. The angles exceed 180 degrees in 2 seconds after the fault is cleared. If the angles exceed 180 degrees, the generators will shut down and the voltage of the whole system will collapse. Fig. 8 shows the proposed TSI and average area voltages response signals due to the fault. The figure shows that the average area voltages fluctuated due to the severe fault. This contingency is considered as a severe contingency because it is said to create transiently unstable state in the power system.

D. Case 4 - A three-phase fault is created at bus 2196 in the Northern Grid and a transmission line connecting bus 2196 and bus 2326 in the same area is disconnected after the fault is cleared at 100 ms. Fig. 8 shows the individual rotor angle plots for each machine in their respected areas (Fig. 9(a) – (e)) and the proposed TSI (Fig. 9(f)). From the plots, it shows that the fault does not affect the system stability since the individual rotor angle plots of all machines in their respective areas do not violate the stability limit of their areas and the power system. The COI-referred rotor angles plots are also consistent with the individual rotor angles plots and they remain in synchronism after the fault has occurred. Fig. 10 shows the proposed TSI and average area voltages response signals due to the disturbances. This also illustrates a transiently stable case as in Case 1.

E. Case 5 - A three-phase fault is created at bus 2510 in the Northern Grid and a transmission line connecting bus 2510 and bus 2511 in the same area is disconnected. Fig. 11 shows the individual rotor angle plots for each machine in their respected areas (Fig. 11(a) – (e)) and the proposed TSI (Fig. 11(f)) when subjected to the fault. It can be seen from Fig. 11(a) – (e) that all the rotor angles in their respective areas increases exponentially and not uniform. The increase in the rotor angles due to the fault implies that the power system become unstable. The COI-referred rotor angles plot as shown in Fig. 11(f) is also consistent with the individual rotor angle plots in which all the areas COI-referred rotor angles go out of step after the fault has occurred. The angles exceed 180 degrees in 2 seconds after the fault is cleared, thus causing the generators to shut down and the voltage of the whole system will collapse. Fig. 12 shows the proposed

TSI and average area voltages response signals due to the fault. It shows that the system becomes unstable after the fault had occurred. This contingency is considered as a severe contingency because it is said to create a transiently unstable state in the power system.

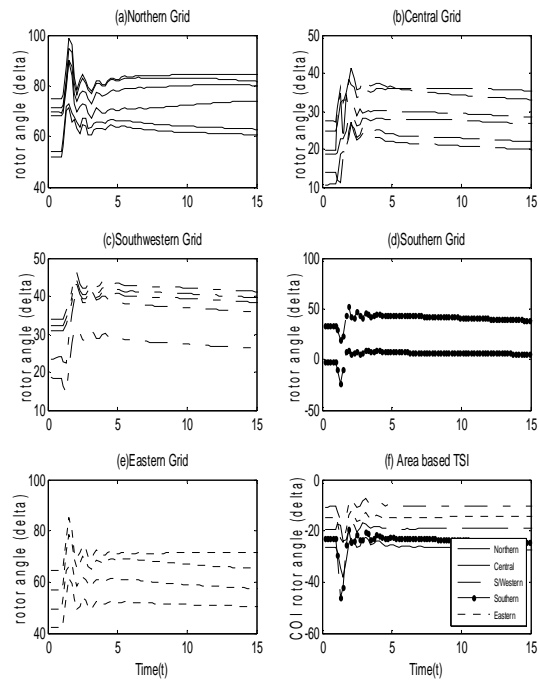


Fig. 3 Case 1: The system remains stable after a fault at bus 73 (Central Grid)

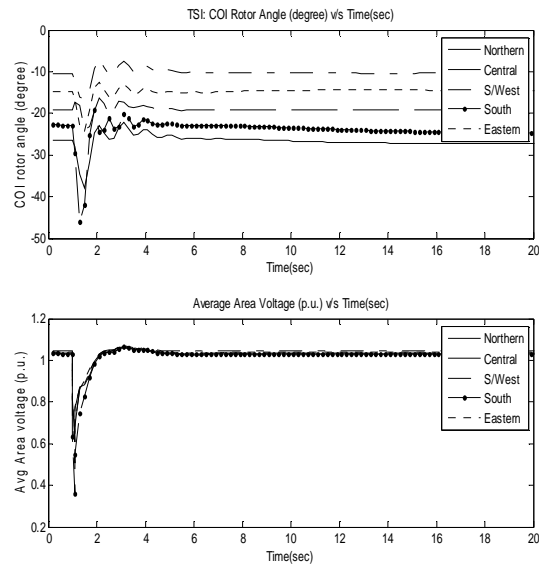


Fig. 4 Case 1: Average area voltages responses in comparison with the proposed TSI

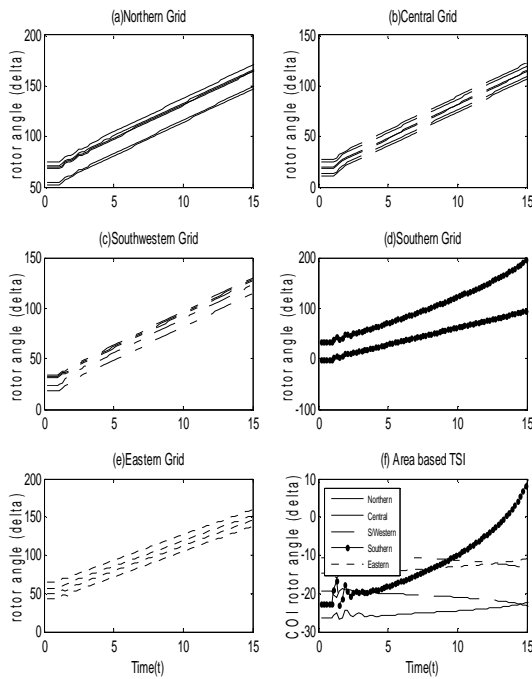


Fig. 5: Case 2: A warning state of the system after a fault at bus 132 (Southern Grid)

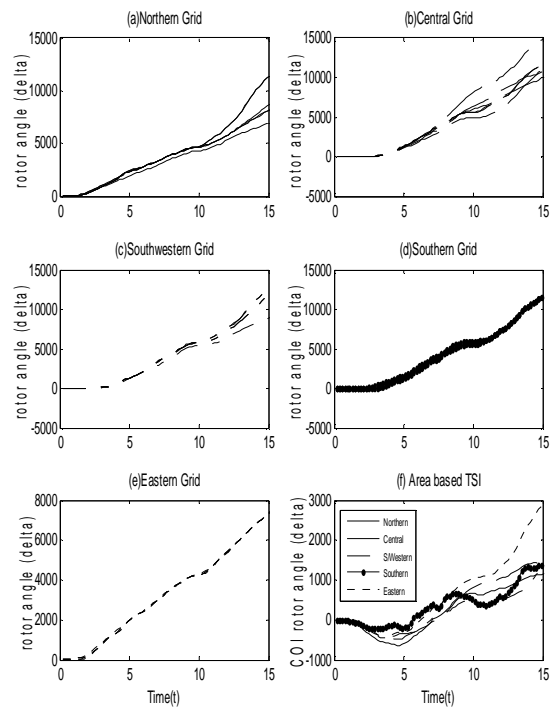


Fig. 7: Case 3: The system becomes unstable after the fault at bus 2118 (Northern Grid)

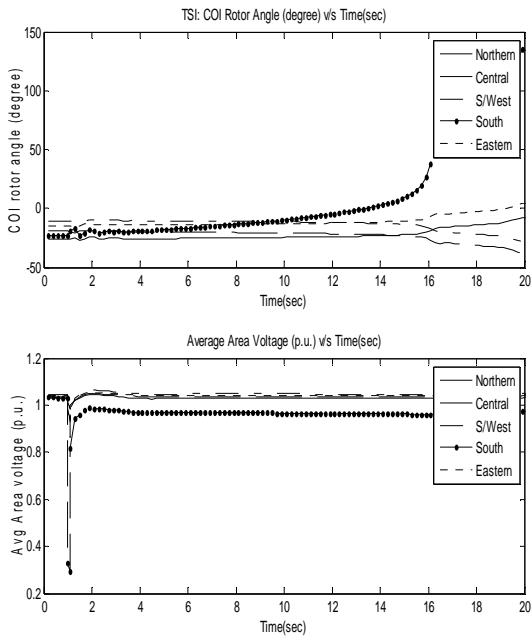


Fig. 6: Case 2: Average area voltages responses in comparison with the proposed TSI

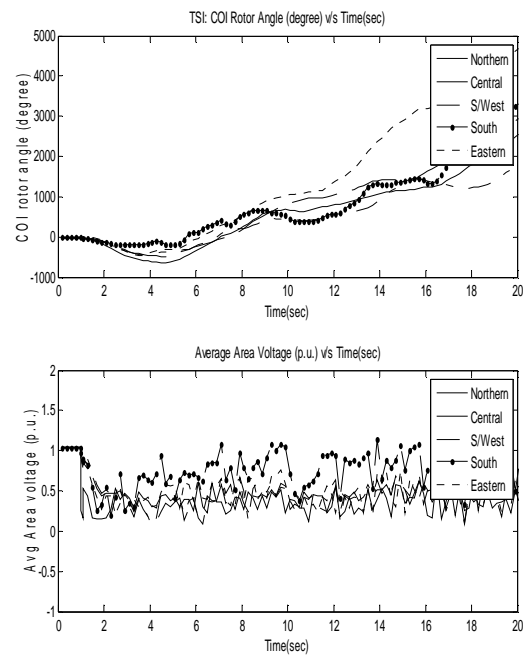


Fig. 8: Case 3: Average area voltages responses in comparison with the proposed TSI

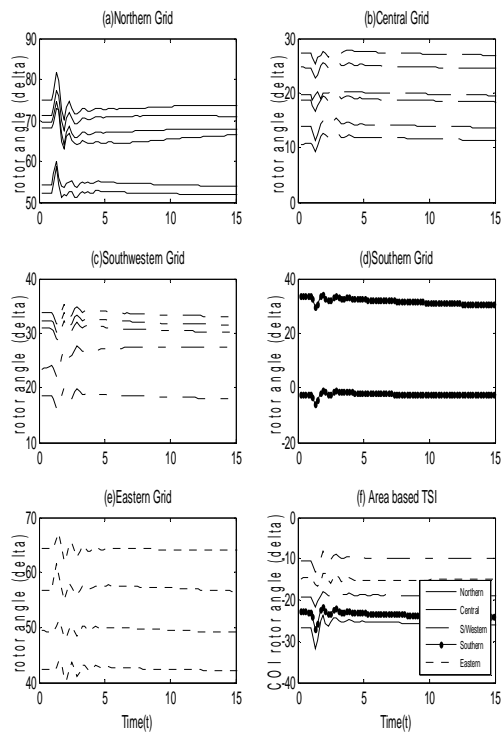


Fig. 9: Case 4: The system remains stable after the fault at bus 2196 (Northern Grid)

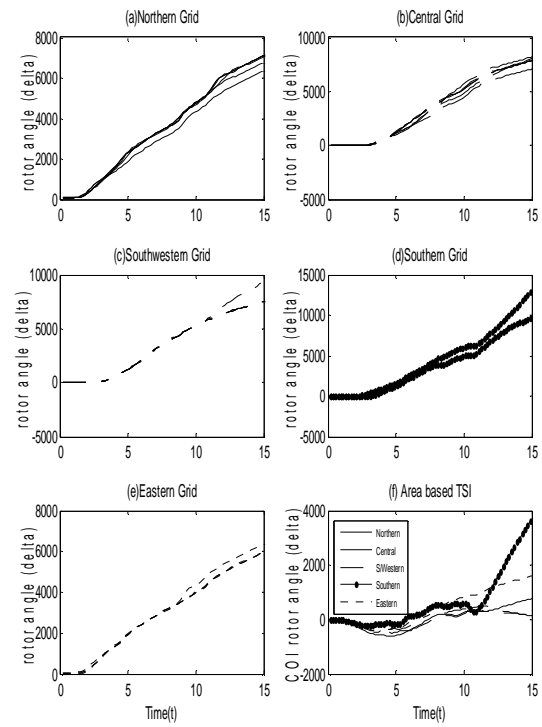


Fig.11: Case 5: The system becomes unstable after the fault at bus 2510 (Northern Grid)

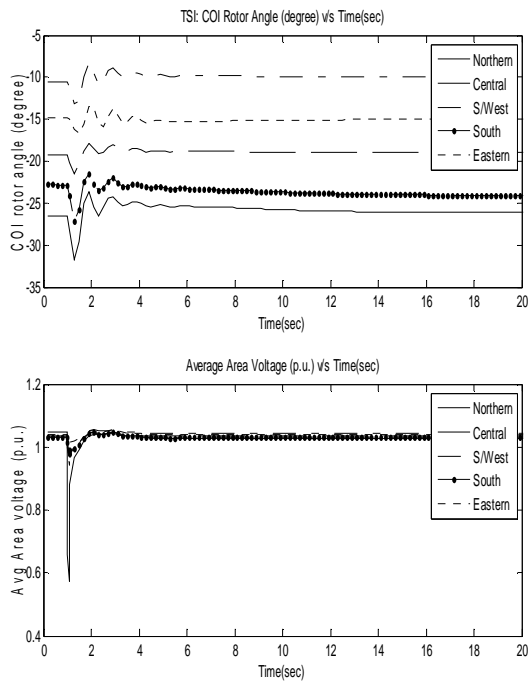


Fig. 10: Case 4: Average area voltages responses in comparison with the proposed TSI

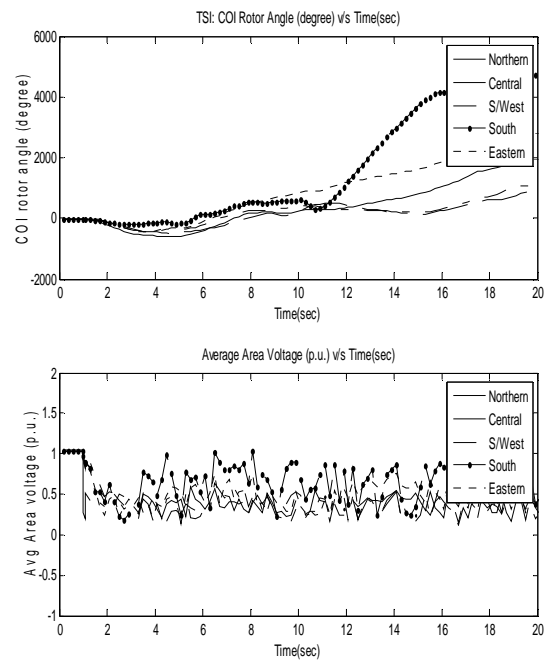


Fig. 12: Case 5: Average area voltages responses in comparison with the proposed TSI

## VII. CONCLUSION

This paper presents a new transient stability index called the Area-based COI-referred Transient Stability Index for a large electrical power system. The proposed index is defined by associating with each area of the power system an equivalent inertia representing the total inertia of the generation located in that area. Through this index, power system operator will need to monitor only five COI-referred rotor angles instead of all 23 rotor angles. Furthermore, the index provides clearly the status of each area and the whole power system due to disturbances. Simulation works on the Malaysia power system enhanced the effectiveness of the proposed index. Application of the proposed index in transient stability assessment is promising.

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## VIII. BIOGRAPHIES



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