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Piecewise Power Flow Solution Using Impedance Parameters



Abstract— Power system engineers are continually faced with the problem of handling large-scale networks arising due to system expansion and greater interconnections. Large-scale systems can be solved using piecewise solution technique wherein the original system is torn into several subsystems, each subsystem is solved independently and the solutions of subsystems are tied up to obtain the total solution. In the proposed method, impedance parameters are consistently used to solve power flow problem of large power systems. The inverse of the Jacobian matrix of each subsystem is derived from the bus impedance matrix of that subsystem. The effect of the cut elements are accounted through proper power injections. The proposed power flow method is successfully tested on IEEE standard systems and found to give good results.

Keywords – Large-scale Power System Problems, Piecewise Method, Power Flow Solution, Impedance Parameters.

I. INTRODUCTION

Power flow problem consist of calculation of bus voltages in a power system for a specified set of bus conditions. The results of power flow study are useful for several operational and planning problems. The Newton Raphson Power Flow (NRPF) method [1] and the Fast Decoupled Power Flow (FDPF) method [2] are used to obtain power flow solutions. Both these methods make use of bus admittance matrix. NRPF method takes only a few iterations. However, for each iteration, lot of computational efforts is needed to compute the elements of the Jacobian matrix and to invert it. FDPF method makes use of constant B and B' matrices that are obtained from the bus admittance matrix. The computer time required for each iteration is much less. However, this method takes more number of iterations for the solution to converge [3,4,5].

Power flow solution can be carried out using impedance parameters [6]. In this method, the inverse of the Jacobian matrix, J_v^{-1} is derived the bus impedance matrix of the transmission network and it is used as a constant matrix in getting the solution. As compared to FDPF method, the power flow solution using impedance parameters takes lesser

number of iterations to get the final solution.

In the power flow method that uses impedance parameters, bus impedance matrix is constructed first. Unlike the bus admittance matrix, the bus impedance matrix is generally full with all non-zero elements. For a large-scale power system, large amount of computer storage is needed to store the bus impedance matrix. With a limited computer memory, power flow solution of large-scale power system becomes a challenging problem.

Piecewise solution is a method used to solve large-scale problems [7]. The large system is torn into several subsystems by removing a few elements from the original system. The removed elements are called cut elements. The mathematical model has a special structure which requires lesser computer memory. Further, this special structure is suitable for piecewise solution procedure.

For a 1000-bus power network, the bus impedance matrix will be of size 1000 x 1000. If the power network is torn into 5 subnetworks of equal size, 5 numbers of 200 x 200 matrices with a few more matrices of smaller size, are to be handled and this needs lesser computer storage. This paper describes the power flow solution of large power systems, using the bus impedance matrices of different subsystems.

II. DEVELOPMENT OF POWER FLOW MODEL

Let the given large power system network is torn into m numbers of subsystems by removing N_c numbers of cut elements. While tearing the network, it is to be ensured that (i) each subnetwork is individually connected (ii) each subnetwork contains the ground bus and (iii) no mutual coupling is present between the subnetworks. In a large practical power system, it is not difficult to adhere with the above requirements [8]. Each subnetwork is also called an area.

In the multi-area case, the real and reactive power flows

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in the cut elements are treated as additional problem variables. These powers are written as a function of the bus voltages and the cut element parameters as

$$P_C = p_C(V, \delta, z_C) \quad (1)$$

$$Q_C = q_C(V, \delta, z_C) \quad (2)$$

where

P_C = vector of real power flows in the cut elements

Q_C = vector of reactive power flows in the cut elements

z_C = impedance vector of cut elements

V = vector of bus voltage magnitudes

δ = vector of bus voltage phase angles.

Each cut element is a link between two subnetworks. The connectivity of the cut elements is shown by the bus incidence matrix. In a N-bus power system, if there are two cut elements, the first one is from bus p to bus q and the second one is from bus r to bus s , the corresponding bus incidence matrix, K is

$$K = \begin{matrix} 1 & \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ \vdots & \vdots \\ p & 1 & 0 \\ \vdots & \vdots & \vdots \\ q & -1 & 0 \\ \vdots & 0 & 0 \\ r & 0 & 1 \\ \vdots & \vdots & \vdots \\ s & 0 & -1 \\ \vdots & \vdots & \vdots \\ N & 0 & 0 \end{bmatrix} \end{matrix} \quad (3)$$

Denoting the variables belonging to the different subsystems by proper subscripts, the power equation constraints are written as

$$\left. \begin{aligned} P(1)\{V(1), \delta(1)\} + K(1)P_C - P_{Net}(1) &= \underline{0} \\ Q(1)\{V(1), \delta(1)\} + K(1)Q_C - Q_{Net}(1) &= \underline{0} \\ \vdots & \\ P(m)\{V(m), \delta(m)\} + K(m)P_C - P_{Net}(m) &= \underline{0} \\ Q(m)\{V(m), \delta(m)\} + K(m)Q_C - Q_{Net}(m) &= \underline{0} \\ p_C(V, \delta) - P_C &= \underline{0} \\ q_C(V, \delta) - Q_C &= \underline{0} \end{aligned} \right\} \quad (4)$$

where

$\underline{0}$ = vector of appropriate size with all elements zero

$P(i)$ = vector of real bus powers flowing out through the transmission network in subsystem i

$Q(i)$ = vector of reactive bus powers flowing out through the transmission network in subsystem i

$K(i)$ = portion of bus incidence matrix corresponding to subsystem i

In the above formulation, the power losses in the cut elements are neglected. Then only P_{c1} and Q_{c1} , real and reactive power in the cut element 1, will be the power that flow out at bus p and $-P_{c1}$ and $-Q_{c1}$ will be the power that flow out at bus

q . The power loss in the cut elements are included while calculating the mismatch powers as discussed later.

In the power equation constraints shown in equation (4), $V(i), \delta(i)$ $i = 1, 2, \dots, m$ and P_C and Q_C are the problem variables. Linearizing this equation and neglecting the higher order terms, we get

$$\begin{bmatrix} \frac{\partial P(1)}{\partial \delta(1)} & \frac{\partial P(1)}{\partial V(1)} \\ \frac{\partial Q(1)}{\partial \delta(1)} & \frac{\partial Q(1)}{\partial V(1)} \\ \vdots & \vdots \\ \frac{\partial P(m)}{\partial \delta(m)} & \frac{\partial P(m)}{\partial V(m)} \\ \frac{\partial Q(m)}{\partial \delta(m)} & \frac{\partial Q(m)}{\partial V(m)} \\ \frac{\partial p_c}{\partial \delta(1)} & \frac{\partial p_c}{\partial V(1)} & \dots & \frac{\partial p_c}{\partial \delta(m)} & \frac{\partial p_c}{\partial V(m)} \\ \frac{\partial q_c}{\partial \delta(1)} & \frac{\partial q_c}{\partial V(1)} & \dots & \frac{\partial q_c}{\partial \delta(m)} & \frac{\partial q_c}{\partial V(m)} \end{bmatrix} \begin{bmatrix} K(1) & [0] \\ [0] & K'(1) \\ \vdots & \vdots \\ K(m) & [0] \\ [0] & K'(m) \\ -U & [0] \\ [0] & -U \end{bmatrix} \begin{bmatrix} \Delta \delta(1) \\ \Delta V(1) \\ \vdots \\ \Delta \delta(m) \\ \Delta V(m) \\ \Delta P_c \\ \Delta Q_c \end{bmatrix} = \begin{bmatrix} \Delta P(1) \\ \Delta Q(1) \\ \vdots \\ \Delta P(m) \\ \Delta Q(m) \\ 0 \\ 0 \end{bmatrix} \quad (5)$$

where

$$\Delta P(i) = P_{Net}(i) - P(i)\{\delta(i), V(i)\} - K(i)P_C \quad (6)$$

$$\Delta Q(i) = Q_{Net}(i) - Q(i)\{\delta(i), V(i)\} - K'(i)Q_C \quad (7)$$

$$[0] = \text{zero matrix of appropriate size} \quad (8)$$

$$U = \text{Unity matrix of appropriate size} \quad (9)$$

The matrix $K'(i)$ differs from the matrix $K(i)$. Suppose that the j th bus in subsystem i is a voltage controlled (P-V) bus, then $K'(i)$ is obtained from $K(i)$ by deleting its j th row.

Necessary deletion of row and column corresponding to the slack and P-V buses must be carried out in the Jacobian matrix of different subsystems. In equation (5)

$$\begin{bmatrix} \frac{\partial P(i)}{\partial \delta(i)} & \frac{\partial P(i)}{\partial V(i)} \\ \frac{\partial Q(i)}{\partial \delta(i)} & \frac{\partial Q(i)}{\partial V(i)} \end{bmatrix} = \text{Jacobian matrix, } J_v(i) \quad (10)$$

$$\begin{bmatrix} K(i) & [0] \\ [0] & K'(i) \end{bmatrix} = B(i) \quad (11)$$

$$\begin{bmatrix} \Delta \delta(i) \\ \Delta V(i) \end{bmatrix} = \Delta x_0(i) \quad (12)$$

$$\begin{bmatrix} \Delta P_c \\ \Delta Q_c \end{bmatrix} = \Delta x_c \quad (13)$$

$$\begin{bmatrix} \Delta P(i) \\ \Delta Q(i) \end{bmatrix} = \Delta S(i) \quad (14)$$

$$J(i)^{-1} = \begin{matrix} & \delta & V \\ P & \begin{bmatrix} X' & -R' \\ R & X \end{bmatrix} \\ Q & \end{matrix} \quad (37)$$

3. Perform Kron's elimination in the region Q-V corresponding to all the voltage controlled buses. The resultant matrix is $J_v(i)^{-1}$.

The corrections on the state variables as given by equation (36) can be computed only if the value of Δx_c is known. Substituting equation (35) in equation (34) we get

$$\Delta x_c = L J_v^{-1} \Delta S - L J_v^{-1} B \Delta x_c \quad (38)$$

From the above

$$\begin{aligned} \Delta x_c &= [U + L J_v^{-1} B]^{-1} L J_v^{-1} \Delta S \\ &= [U + \sum_{i=1}^m L(i) J_v(i)^{-1} B(i)]^{-1} \sum_{i=1}^m L(i) J_v(i)^{-1} \Delta S(i) \end{aligned} \quad (39)$$

In any iteration, for the calculated mismatch power vector ΔS , the vector Δx_c is first computed from equation (40) and then the vector Δx_0 is calculated from equation (36).

It is to be observed that the elements of L matrix in equation (16) are voltage dependent. Since there are only a few non-zero elements, it will not be difficult to evaluate the L matrix in each iteration.

While formulating the problem, the power loss in the cut elements are assumed as zero. However, while computing the calculated powers, cut element powers at the sending end and the receiving end are evaluated and added to the power flows out in each bus through all the connected transmission lines. This will enable us to calculate the correct values of mismatch powers.

IV. SOLUTION PROCEDURE

The following solution procedure is followed to carry out the power flow solution of large power system that has been torn into smaller subsystems.

Step 1: For each subsystem, read the element data and bus data; read the cut element data.

Step 2: Construct the bus impedance matrix of the subsystems and obtain $J_v(i)^{-1}$ matrices. For each subsystem compute $B(i)$ matrix.

Step 3: For each subsystem, calculate the line flows and compute the reactive generations at the P-V buses and check for switching of bus type; if there is switching, suitably recalculate $J_v(i)^{-1}$ matrix [6].

Step 4: From the calculated line flows and the power flows in the cut elements, compute the mismatch powers. If convergence is reached, go to step 6; otherwise continue.

Step 5: Compute the elements of L matrix. Calculate Δx_c from equation (40). For each subsystem, determine $\Delta x_0(i)$ from equation (36) and update the bus voltages. Go to step 3.

Step 6: Compute the line flows, bus powers and line losses, if necessary. Print the results and stop the computation.

V. RESULTS

The developed method is tested on IEEE systems using MATLAB. The results for the IEEE 14-bus system are presented. Initially the system is kept as one unit as shown in Fig. 1.

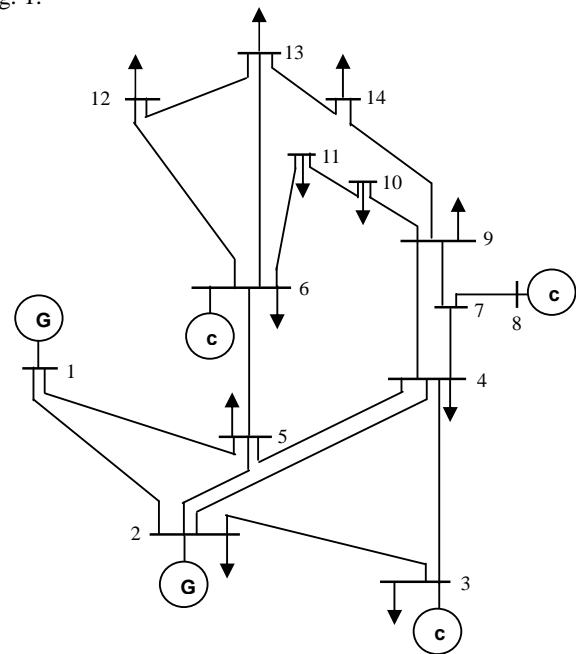


Fig. 1 IEEE 14-bus single unit system

Power flow solution is obtained using impedance parameters [6], wherein J_v^{-1} matrix, which is derived from the bus impedance matrix, played a key role in getting the solution.

The IEEE 14-bus system is torn into two subsystems by removing the elements 4-7, 4-9 and 5-6. The resultant two area system is shown in Fig. 2. Area 1 contains buses 1 to 5 and includes 7 elements. Buses 6 to 14 with 10 elements form area 2.

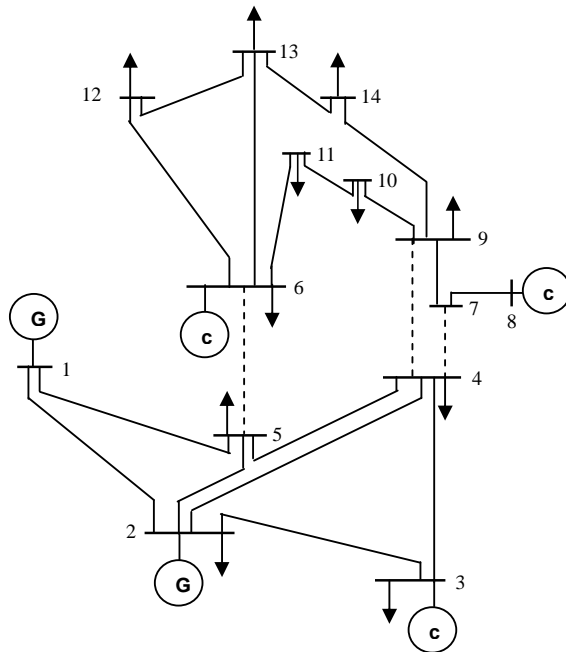


Fig. 2 IEEE 14-bus two area system

Power flow solution of this two area system is carried out following the procedure presented. The final solution exactly agrees with that of single area system. The results are shown in Table I.

TABLE I
BUS VOLTAGES

| Bus No. | Single Area System | Two Area System |
|---------|--------------------|-----------------|
| 1 | 1.0600∠0 | 1.0600∠0 |
| 2 | 1.0450∠-4.9853 | 1.0450∠-4.9853 |
| 3 | 1.0100∠-12.7395 | 1.0100∠-12.7395 |
| 4 | 1.0155∠-10.2836 | 1.0155∠-10.2836 |
| 5 | 1.0183∠-8.7599 | 1.0183∠-8.7599 |
| 6 | 1.0700∠-14.2183 | 1.0700∠-14.2183 |
| 7 | 1.0605∠-13.3377 | 1.0605∠-13.3377 |
| 8 | 1.0900∠-13.3377 | 1.0900∠-13.3377 |
| 9 | 1.0550∠-14.9184 | 1.0550∠-14.9184 |
| 10 | 1.0502∠-15.0801 | 1.0502∠-15.0801 |
| 11 | 1.0565∠-14.7801 | 1.0565∠-14.7801 |
| 12 | 1.0551∠-15.0725 | 1.0551∠-15.0724 |
| 13 | 1.0502∠-15.1511 | 1.0502∠-15.1510 |
| 14 | 1.0349∠-16.0210 | 1.0349∠-16.0210 |

The number of iterations taken is shown in Table II.

TABLE II
NUMBER OF ITERATIONS

| Single Area System | Two Area System |
|--------------------|-----------------|
| 9 | 14 |

VI. CONCLUSIONS

It is well known that the bus impedance matrix is successfully used in short circuit analysis. To be in line with this, it is better to use the impedance parameters for the power flow solution also. In the proposed method, bus impedance matrices of the subsystems are effectively used in getting the solution.

From the results shown above, it is demonstrated that the multi-area power solution procedure discussed in this paper, gives exactly same results as that of single area system. As compared to single area system, multi-area system is expected to take more number of iterations.

Multi-area system will require lesser computer storage. The study on very large power systems will reveal the actual saving in computer storage. When the system size exceeds beyond certain level, solving it as a single unit is not possible because of excessive computer storage requirement. In that situation, multi-area solution is the answer.

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X. BIOGRAPHIES



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