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Power Flow Solution Using Impedance Parameters

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Abstract— Bus admittance matrix is invariably used for the power flow solution. In this paper, power flow analysis that makes use of only impedance parameters is presented. The inverse of Jacobian matrix (J_v^{-1}) is obtained very easily by doing simple mathematical operations on the bus impedance matrix of the transmission network. This constant J_v^{-1} matrix is used to update the solution in each iteration. The proposed method is tested on IEEE systems and found to give good results. As compared to the Fast Decoupled Power Flow method, the proposed one takes lesser number of iterations to get the exact solution. The proposed power flow method can be extended to solve power flow problem associated with large power systems, using piecewise solution technique.

Keywords – Power Flow Solution, Impedance Parameters, Faster Solution.

I. INTRODUCTION

Power flow analysis is the most vital study carried out by the power companies. It aims to calculate the bus voltages for a specified set of bus power injection. The mathematical model being a set of non-linear algebraic equations, iterative procedure is followed to get the solution. At first Gauss-Seidel iterative procedure was introduced for the power flow solution [1]. Though this procedure is simple, it is not recommended because of poor convergence characteristic. Followed by this, Newton-Raphson (NR) method [2] was introduced. Exact problem formulation and good convergence characteristic are the merits of this method. In each iteration, the elements of the Jacobian matrix are to be calculated and the inverse of the Jacobian matrix is to be obtained. The amount of calculation involved in each iteration limits the use of NR method for power flow analysis for on-line applications. Fast Decoupled Power Flow (FDPF) method, that was introduced subsequently [3], uses the constant B and $B^{'}$ matrices. Computationally this method is simple. But a number of assumptions are made to make B and $B^{'}$ as constant matrices. Particularly, line resistances are assumed to be much less as compared to their reactances while forming B and $B^{'}$ matrices. FDPF method takes more number of iterations to get the final solution.

Most of the power flow methods make use of the bus admittance matrix simply because this matrix can be assembled easily. In practice, the series parameters of the power system components are specified in impedance. Further, when system model is developed in impedance frame, the voltage solution can be obtained much faster. Short circuit analysis employing the bus impedance matrix is a standing example. It will be advantageous if the power flow analysis also could be conducted using bus impedance matrix.

It may be noted that the matrices B and $B^{'}$ used in FDPF method are of admittance in nature. These matrices are inverted and multiplied with the error vector to find the changes in the bus voltage magnitudes and phase angles. Thus the inverse of the Jacobian matrix is of impedance in nature [4].

In this paper, the inverse of the Jacobian matrix is obtained from the bus impedance matrix of the transmission network by performing a few simple mathematical operations. This constant inverse of the Jacobian matrix is used to compute the changes in the bus voltage magnitudes and phase angles. This formulation includes the resistances of transmission lines while forming the constant J_v^{-1} matrix, resulting in substantial reduction in the number of iterations. The developed method is successfully tested on IEEE systems to prove its validity.

II. FUNDAMENTALS NECESSARY TO DEVELOP POWER FLOW MODEL

The performance of the transmission network can be written either in admittance form or impedance form as

$$Y_{Bus}\, V_{Bus} = I_{Bus} \tag{1}$$

$$Z_{Bus}\, I_{Bus} = V_{Bus} \tag{2}$$

In expanded form we have

$$\begin{bmatrix} Y_{11} & Y_{12} & \cdots & Y_{1N} \\ Y_{21} & Y_{22} & \cdots & Y_{2N} \\ \vdots & \vdots & \vdots & \vdots \\ Y_{N1} & Y_{N2} & \cdots & Y_{NN} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ \vdots \\ V_N \end{bmatrix} = \begin{bmatrix} I_1 \\ I_2 \\ \vdots \\ I_N \end{bmatrix} \tag{3}$$

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$$\begin{bmatrix} Z_{11} & Z_{12} & \cdots & Z_{1N} \\ Z_{21} & Z_{22} & \cdots & Z_{2N} \\ \vdots & \vdots & \vdots & \vdots \\ Z_{N1} & Z_{N2} & \cdots & Z_{NN} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ \vdots \\ I_N \end{bmatrix} = \begin{bmatrix} V_1 \\ V_2 \\ \vdots \\ V_N \end{bmatrix} \quad (4)$$

Without loss of generality bus 1 is taken as the slack bus. Then we delete the first row and first column of Y_{Bus} matrix and the resultant matrix, Y'_{Bus}, given by

$$Y'_{Bus} = \begin{bmatrix} Y_{22} & Y_{23} & \cdots & Y_{2N} \\ Y_{32} & Y_{33} & \cdots & Y_{3N} \\ \vdots & \vdots & \vdots & \vdots \\ Y_{N2} & Y_{N3} & \cdots & Y_{NN} \end{bmatrix} \quad (5)$$

is useful in power flow calculations.

The corresponding Z'_{Bus} matrix, which is the inverse of Y'_{Bus} matrix, is obtained from Z_{Bus} by performing Kron's elimination with respect to the first row and first column taking Z_{11} as the pivotal element. Thus

$$Z'_{ij} = Z_{ij} - \frac{Z_{i1}\ Z_{1j}}{Z_{11}} \quad \text{for i and j = 2,3,…,N} \quad (6)$$

The above can be generalized for any two matrices that are inverse of each other. Kron's elimination performed on Z_{Bus} ,on row i and column i with ii as pivotal element can also be thought of grounding bus i and finding the modified bus impedance matrix.

Separating the real and imaginary parts, matrices Y'_{Bus} and Z'_{Bus} are written as

$$Y'_{Bus} = G' + j\ B' \quad (7)$$

$$Z'_{Bus} = R' + j\ X' \quad (8)$$

Then $[G' + j\ B'][R' + j\ X'] = [U]$ (9)

Therefore $[G'R' - B'X'] = [U]$ (10)

$[B'R' + G'X'] = [0]$ (11)

III. DERIVATION OF J_v^{-1} MATRIX

In Newton-Raphson method, we have

$$\begin{bmatrix} J_1 & J_2 \\ J_3 & J_4 \end{bmatrix} \begin{bmatrix} \Delta\delta \\ \Delta V \end{bmatrix} = \begin{bmatrix} \Delta P \\ \Delta Q \end{bmatrix} \quad (12)$$

i.e. $$J \begin{bmatrix} \Delta\delta \\ \Delta V \end{bmatrix} = \begin{bmatrix} \Delta P \\ \Delta Q \end{bmatrix} \quad (13)$$

Here J is the Jacobian matrix. Changes in the state variables namely $\Delta\delta$ and ΔV are obtained from

$$\begin{bmatrix} \Delta\delta \\ \Delta V \end{bmatrix} = J^{-1} \begin{bmatrix} \Delta P \\ \Delta Q \end{bmatrix} \quad (14)$$

Elements of J_1, J_2, J_3 and J_4 matrices are calculated from the standard expressions for the diagonal and off-diagonal elements [5]. The following assumptions are now made.

1. All the bus voltages are equal to $1\angle 0^0$ p.u.
2. The power system is in base case condition.
3. There is no shunt element at any bus.

This really means that the Jacobian is computed at the base case point neglecting the shunt elements at the buses. Under this condition, the Jacobian matrix becomes

$$J = \begin{bmatrix} -B' & G' \\ -G' & -B' \end{bmatrix} \quad (15)$$

where G' and B' are as given in equation (7). The inverse of the above Jacobian matrix is

$$J^{-1} = \begin{bmatrix} X' & -R' \\ R' & X' \end{bmatrix} \quad (16)$$

The above result can be verified by checking that

$$\begin{bmatrix} -B' & G' \\ -G' & -B' \end{bmatrix} \begin{bmatrix} X' & -R' \\ R' & X' \end{bmatrix} = \begin{bmatrix} U & 0 \\ 0 & U \end{bmatrix} \quad (17)$$

when eqns. (10) and (11) are used.

It is to be noted that J^{-1} matrix as given in equation (16) can be assembled, once Z'_{Bus} matrix is computed using equation (6). When the J^{-1}, inverse of the Jacobian matrix is known, changes in state variables are calculated from equation (14).

The presence of voltage controlled (P-V) buses calls for certain modification on the J^{-1} matrix as discussed below. Equations (13) and (14) are written as

$$\begin{bmatrix} -B' & G' \\ -G' & -B' \end{bmatrix} \begin{bmatrix} \Delta\delta \\ \Delta V \end{bmatrix} = \begin{bmatrix} \Delta P \\ \Delta Q \end{bmatrix} \quad (18)$$

$$\begin{bmatrix} \Delta\delta \\ \Delta V \end{bmatrix} = \begin{bmatrix} X' & -R' \\ R' & X' \end{bmatrix} \begin{bmatrix} \Delta P \\ \Delta Q \end{bmatrix} \quad (19)$$

In any voltage controlled bus, reactive power specification is not specified and the voltage magnitude is specified. These are to be accommodated in the eqns.(18) and (19). Suppose bus k is a voltage controlled bus. Then, the equation corresponding to ΔQ_k is to be deleted and ΔV_k must be removed from the list of variables. This amounts to deletion of one row and one column in the J matrix, $\begin{bmatrix} -B' & G' \\ -G' & -B' \end{bmatrix}$ in equation (18). The corresponding change in J^{-1} matrix, $\begin{bmatrix} X' & -R' \\ R' & X' \end{bmatrix}$ in equation (19) is carried out, by using Kron's elimination. This process need to be repeated for all the voltage controlled buses. The resultant J^{-1} matrix is called as J_v^{-1} matrix. Once the J_v^{-1} matrix is computed, changes in voltage magnitudes and voltage phase angles are calculated as

$$\begin{bmatrix} \Delta\delta \\ \Delta V \end{bmatrix} = J_V^{-1} \begin{bmatrix} \Delta P \\ \Delta Q \end{bmatrix} \quad (20)$$

Normally, at the voltage controlled buses, the generator reactive powers are to be kept within certain limits. At every iteration, the generator reactive powers at these buses are to be calculated and checked against the limits. If at any bus the reactive power violates the limits given, then the bus type is switched from P-V to P-Q bus with the reactive generation set at the appropriate limit Q_{Min} or Q_{Max}. When such a switching occurs, J_V^{-1} matrix has to be recalculated starting from J^{-1} matrix. Thereafter, checks are to be made to see whether the reactive generation backs off the limit again; if this occurs, then the bus type should be switched back from P-Q to P-V and the J_V^{-1} matrix has to be recalculated from the last J_V^{-1} matrix available.

IV. SOLUTION PROCEDURE

The solution procedure for the power flow study is summarized below.

Step 1: Read the element (Transmission line and transformer) data and bus data. Assume flat start. Construct Z_{Bus} matrix of the network using the bus impedance building algorithm [6]. Once knowing the slack bus, calculate the Z'_{Bus} matrix using Kron's elimination. Assemble J^{-1} matrix as shown in equation (16) and store it. Knowing the voltage controlled buses, compute J_V^{-1} matrix using Kron's elimination.

Step 2: Calculate the line flows. Compute the reactive generations at the voltage controlled buses and check whether switching of the bus type occurs. If switching occurs, go to step 3; otherwise go to step 4.

Step 3: If there is switching from P-V bus to P-Q bus, calculate J_V^{-1} matrix starting from J^{-1} matrix. If there is a switching back from P-Q bus to P-V bus, simply recalculate J_V^{-1} matrix from the latest J_V^{-1} matrix available.

Step 4: From the calculated line flows, compute the mismatch powers. Check whether the convergence is achieved. If the convergence is reached, go to step 6: otherwise continue.

Step 5: Calculate $\Delta\delta$ and ΔV from equation (20) and update the bus voltages. Go to step 2.

Step 6: Compute the line flows, bus powers and line losses if necessary. Print the results and stop the computation.

V. SPECIAL ASPECTS

1. In the above stated procedure only impedance parameters are used in all stages. Generally in NR method as well as FDPF method, elements of bus admittance matrix are used while computing the calculated powers. In the work presented in this paper, only impedance parameters are used to compute the calculated powers as discussed below.

The element considered in power flow analysis may be a transmission line or a transformer between buses i and j. A transmission line is represented by the series impedance $r_{ij} + j\,x_{ij}$ and equal shunt admittance $g_{sh} + j\,b_{sh}$ at the receiving and sending ends. A transformer is represented by the series impedance $r_{ij} + j\,x_{ij}$ and off-nominal turns ratio *a* with tap setting facility at bus *i*. For such a transmission line / transformer, taking the bus voltages as $V_i \angle\delta_i$ and $V_j \angle\delta_j$, the real and reactive power flows are computed as follows:
Defining

$$g_{ij} = r_{ij} / (r_{ij}^2 + x_{ij}^2) \quad (21)$$

$$b_{ij} = -x_{ij} / (r_{ij}^2 + x_{ij}^2) \quad (22)$$

$$\delta_{ij} = \delta_i - \delta_j \quad (25)$$

we get

$$P_{ij} = V_i^2 \left(\frac{g_{ij}}{a^2} + g_{sh}\right) - \frac{V_i V_j}{a}(g_{ij} \cos\delta_{ij} + b_{ij} \sin\delta_{ij}) \quad (26)$$

$$Q_{ij} = -V_i^2 \left(\frac{b_{ij}}{a^2} + b_{sh}\right) - \frac{V_i V_j}{a}(g_{ij} \sin\delta_{ij} - b_{ij} \cos\delta_{ij}) \quad (27)$$

$$P_{ji} = V_j^2 (g_{ij} + g_{sh}) - \frac{V_i V_j}{a}(g_{ij} \cos\delta_{ij} - b_{ij} \sin\delta_{ij}) \quad (28)$$

$$Q_{ji} = -V_j^2 (b_{ij} + b_{sh}) + \frac{V_i V_j}{a}(g_{ij} \sin\delta_{ij} + b_{ij} \cos\delta_{ij}) \quad (29)$$

For transmission line, *a* is set to 1 and for transformer g_{sh} and b_{sh} are set to zero.

Calculated real and reactive powers at bus k are computed from

$$P_k = \sum_{\text{for all } i\text{'s connected to } k} P_{ki} \quad (30)$$

$$Q_k = \sum_{\text{for all } i\text{'s connected to } k} Q_{ki} \quad (31)$$

2. In the transmission network, generally there will be a few shunt parameters connected between the bus and the ground. If so, the bus impedance matrix, Z_{Bus} of the transmission network (with ground as reference) can be constructed without any difficulty. On the other hand, if there is no shunt parameter in the transmission network, the ground bus is isolated and Z_{Bus} matrix does not exist. In such a special case, shunt impedance of the order –j200 p.u. may be introduced at a bus other than the slack bus to ensure the existence of the bus impedance matrix Z_{Bus}. This fictitious element can be removed once the Z'_{Bus} matrix is formed.

VI. RESULTS

The developed power flow method is tested on several standard systems to check its validity. It is meaningful to compare the proposed method with the two well known methods namely Newton-Raphson Power Flow (NRPF) method and the Fast Decoupled Power Flow (FDPF) method. The results obtained on the IEEE 14-bus, IEEE 30-bus and IEEE 57-bus systems are given in the following Tables.

TABLE I
BUS VOLTAGES (IEEE 14-BUS SYSTEM)

Bus No.	Newton-Raphson Method	Fast Decoupled Method	Proposed Method
1	$1.0600\angle 0$	$1.0600\angle 0$	$1.0600\angle 0$
2	$1.0450\angle -4.9853$	$1.0450\angle -4.9853$	$1.0450\angle -4.9853$
3	$1.0100\angle -12.7395$	$1.0100\angle -12.7395$	$1.0100\angle -12.7395$
4	$1.0155\angle -10.2836$	$1.0155\angle -10.2836$	$1.0155\angle -10.2836$
5	$1.0183\angle -8.7599$	$1.0183\angle -8.7599$	$1.0183\angle -8.7599$
6	$1.0700\angle -14.2183$	$1.0700\angle -14.2183$	$1.0700\angle -14.2183$
7	$1.0605\angle -13.3377$	$1.0605\angle -13.3377$	$1.0605\angle -13.3377$
8	$1.0900\angle -13.3377$	$1.0900\angle -13.3377$	$1.0900\angle -13.3377$
9	$1.0550\angle -14.9184$	$1.0550\angle -14.9184$	$1.0550\angle -14.9184$
10	$1.0502\angle -15.0801$	$1.0502\angle -15.0801$	$1.0502\angle -15.0801$
11	$1.0565\angle -14.7800$	$1.0565\angle -14.7800$	$1.0565\angle -14.7801$
12	$1.0551\angle -15.0724$	$1.0551\angle -15.0725$	$1.0551\angle -15.0725$
13	$1.0502\angle -15.1510$	$1.0502\angle -15.1510$	$1.0502\angle -15.1511$
14	$1.0349\angle -16.0210$	$1.0349\angle -16.0210$	$1.0349\angle -16.0210$

TABLE II
COMPARISON ON IEEE 14-BUS SYSTEM

	Newton-Raphson Method	Fast Decoupled Method	Proposed Method
No. of iterations	3	28	9
Max. error	9.2413×10^{-8}	8.6210×10^{-6}	6.9127×10^{-6}
CPU time	0.0915 sec	0.1129 sec	0.0971 sec

TABLE III
COMPARISON ON IEEE 30-BUS SYSTEM

	Newton-Raphson Method	Fast Decoupled Method	Proposed Method
No. of iterations	3	24	8
Max. error	7.5490×10^{-7}	7.0951×10^{-6}	7.6210×10^{-6}
CPU time	0.1317 sec	0.1438 sec	0.1375 sec

TABLE IV
COMPARISON ON IEEE 57-BUS SYSTEM

	Newton-Raphson Method	Fast Decoupled Method	Proposed Method
No. of iterations	3	42	17
Max. error	8.2050×10^{-7}	7.1248×10^{-6}	8.4351×10^{-6}
CPU time	0.2031 sec	0.5104 sec	0.4984 sec

The specification of the computer used for the CPU time is 504 MB of Ram and the speed of CPU is Pentium(R) 3.40 GHz. Bus voltages computed for the IEEE 30-bus system and IEEE 57-bus system, by the three methods are all equal, similar to the results for the IEEE 14-bus system.

VII. CONCLUSIONS

In NRPF method the elements of Jacobian matrix are voltage-dependent. Each iteration requires more computational effort. Use of this method on large systems for on-line study will result in computational problems.

In FDPF method, assumptions are made to make the elements of Jacobian matrix independent of bus voltages. While forming the constant matrices B and B' it is assumed that the resistances of transmission lines and transformers are much less as compared to their reactances. These constant matrices are inverted once. In each iteration they are multiplied with the corresponding mismatch power vector to get the improved solution. FDPF method has poor convergence characteristic [5,6].

Since short circuit study is conducted using bus impedance matrix, it is better to carry out power flow analysis also using the bus impedance matrix.

In the proposed method the inverse of the Jacobian matrix is obtained from the bus impedance matrix of the transmission network. The resistances and reactances of transmission lines and transformers are taken as they are. The inverse of the Jacobian matrix is computed at base case point neglecting the shunt elements at the buses. In each iteration, the constant Jacobian inverse matrix J_V^{-1} is multiplied with the mismatch power vector to get the correction needed in the bus voltages. Since the matrix J_V^{-1} is better than the inverse of B and B' matrices, the proposed method takes lesser number of iterations to get the exact solution. The CPU time for the proposed method is less than that for fast decoupled method. The results shown above indicate the merits of the proposed method.

Piecewise solution technique can be used to solve large-scale power system problems. The power flow method presented in this paper can be extended to solve power flow problem of large power systems using piecewise solution technique. This work is reported in another paper.

VIII. ACKNOWLEDGEMENT

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