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## Fuzzy Logic Application in Evaluation of DGA Interpretation Methods



**Abstract:** Dissolved gas-in-oil analysis (DGA) is one of the most useful techniques to detect the incipient faults in large oil-filled transformers. Various methods have been developed to interpret DGA results. Among them are the Key Gas, Rogers Ratio, Logarithmic Nomograph, Doernenburg, IEC Ratio and Duval Triangle. This paper used the DGA data from 69 different cases to test the accuracy and consistency of these methods in interpreting the transformer condition. The key gases considered for evaluation are hydrogen, methane, ethane, ethylene and acetylene. MATLAB programs with and without using Fuzzy logic were developed to automate the evaluation of each method. The difference on accuracy and consistency of each method using and not using Fuzzy logic is presented.

**Index Terms—** DGA interpretation method, Fuzzy Logic, fault gases.

### I. INTRODUCTION

The DGA methods have been employed widely in the transformer industry for condition assessment. Stemming from breakdown or decomposition of the insulation oil, cellulose or paper, gases at various concentrations, such as hydrogen ( $H_2$ ), methane ( $CH_4$ ), ethane ( $C_2H_6$ ), ethylene ( $C_2H_4$ ), acetylene ( $C_2H_2$ ), carbon monoxide (CO), and carbon dioxide ( $CO_2$ ), may be released and partly dissolved in the oil. The causes of the breakdown or decomposition of the insulation material are attributed to electrical and thermal stresses in the transformer. Principles have been developed in the DGA method for the judgment of the fault conditions according to the results from gas chromatographic analysis. Currently there are several methods developed to do the interpretation of the fault type from the dissolved gas data. In this paper, the six methods of interpretation of the fault gases of mineral oil are investigated and compared. They are: Key Gas, Rogers Ratio, Doernenburg, Logarithmic Nomograph, IEC Ratio and Duval Triangle. The study was done to evaluate the

accuracy of each method in predicting the fault and the consistency of each method.

These methods commonly use the multiple numeric thresholds and gas ratio boundaries to classify features of the dissolved gas data as to membership in various intervals. The interval membership information is used to infer a diagnosis. When a fault is intermittent or of low intensity, some of the input features may fall near but outside the expected intervals, with the result that no diagnosis is obtained. There is a possibility to smooth these thresholds and ratio boundaries by using Fuzzy Logic [1-3].

Fuzzy Logic is known as one of the expert systems that can be used to diagnose the faults because of its ability in storing knowledge and using it to make decision [4]. Here, the final diagnosis rules are automatically determined and the membership functions of the corresponding fuzzy subsets are simultaneously adjusted. This can give better judgment on the diagnosis of transformer faults. In this paper, evaluation of DGA methods was firstly done using only basic coding and construction of Simulink block diagram. Then the same set of DGA data was evaluated using a Fuzzy Logic controller. The results are then analyzed and compared.

### II. TESTING METHOD

The data comprises 69 sets of 5 fault gases, obtained from published papers [4-7]. These were used to test each method. The five key gases are  $H_2$ ,  $CH_4$ ,  $C_2H_6$ ,  $C_2H_4$  and  $C_2H_2$ . Table 1 shows the set of data used and fault code for each type of faults used in this paper.

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TABLE 1:  
SET OF DATA USED IN ANALYSIS

Fault Type	Fault Type Code	Number of cases
Thermal fault at low temperature	F <sub>1</sub>	1
Overheating and sparking	F <sub>2</sub>	33
Arcing	F <sub>3</sub>	22
Partial Discharge and Corona	F <sub>4</sub>	8
Normal	F <sub>5</sub>	5

The testing method should be the same for each DGA interpretation method in order to compare their accuracy and consistency. Each method diagnosis was grouped according to the faults type code for comparison. This is shown in Table 2.

TABLE 2:  
GROUPING FOR FAULT TYPE CODES

Method	F <sub>1</sub>	F <sub>2</sub>	F <sub>3</sub>	F <sub>4</sub>	F <sub>5</sub>
Roger	Slight overheating <150°C  Overheating 150°C-200°C  Overheating 200°C-300°C	Conductor overheating  Winding circulating current  Core/tank circulating current.	Flashover.  Arcing  Continuous sparking.	PDs  PDs with tracking	Normal
IEC	Thermal fault <150°C  Thermal fault 150°C-300°C	Thermal fault 300°C-700°C  Thermal fault > 700°C	Discharge of low energy  Discharge of high energy	PDs of low energy density  PDs of high energy density	Normal
Nomograph	Heating	Heating and Discharge	Arcing  Arcing and heating	Arcing, heating and discharge  Arcing and discharge	Normal (<L1)
Doernenburg	Thermal decomposition with very high ratio 4	Thermal decomposition	Arcing	Corona	Normal (< L1)
Duval	Thermal fault <300°C	Thermal fault 300°C-700°C  Thermal fault > 750°C	Low energy discharge  High energy discharge	PDs  Mix thermal and electrical faults	Normal (< L1)
Key Gas	Principal gas: CH <sub>4</sub> and C <sub>2</sub> H <sub>6</sub>	Principal gas: C <sub>2</sub> H <sub>4</sub>	Principal gas: C <sub>2</sub> H <sub>2</sub>	Principal gas: H <sub>2</sub>	Normal (< L1)

A. Basic coding and Simulink diagram (Without Fuzzy System)

The comparison for each method was done using MATLAB programming. A program was developed to run the test based on each method rules for diagnosing the faults. This involved several coding and Simulink block diagrams to run the test. An example is shown in Figure 1. In general, the diagrams consist of three main sections. The first section is for checking the limit value of the fault gases if applicable. The second section is for calculating the ratio and finding the ratio coding if applicable. The last section provides the diagnosis based on the ratio coding sequence or ratio value or fault gas value.

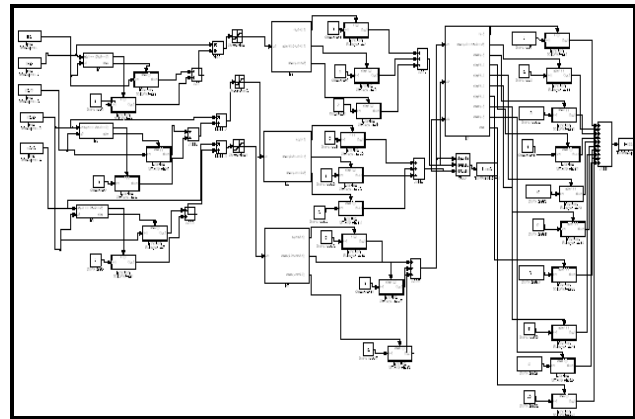


Fig 1: Example of Simulink block diagram developed for testing.

B. Fuzzy diagnosis systems

i. Roger's Ratio Fuzzy System

This fuzzy system consists of 4 ratio codes as inputs. The output comprises 13 interpretation results based on the 11 fault types shown in Table 3 plus one 'normal' and one 'no prediction' to cover those ratio code sequences not included in the table.

TABLE 3:

CLASSIFICATION OF FAULTS BASED ON ROGER'S RATIO CODES[5]

i	j	k	l	Diagnosis
0	0	0	0	Normal deterioration
5	0	0	0	Partial discharge
1-2	0	0	0	Slight overheating <150° C
1-2	1	0	0	Overheating 150° C-200° C
0	1	0	0	Overheating 200° C-300° C
0	0	1	0	General conductor overheating
1	0	1	0	Winding circulating currents
1	0	2	0	Core and tank circulating currents, overheated joints
0	0	0	1	Flashover without power follow through
0	0	1-2	1-2	Arc with power follow through
0	0	2	2	Continuous sparking to floating potential
5	0	0	1-2	Partial discharge with tracking (note CO)

The 4 ratios are classified as either Low (Lo), Medium (Med), High (Hi) or Very High (Vhi) according to membership intervals as defined below:

$$l = C_2H_2 / C_2H_4 = \{Lo, Med, Hi\}$$

$$i = CH_4 / H_2 = \{Lo, Med, Hi, Vhi\}$$

$$k = C_2H_4 / C_2H_6 = \{Lo, Med, Hi\}$$

$$j = C_2H_6 / CH_4 = \{Lo, Hi\}$$

$$i = \begin{cases} 5 & Lo & U < 0.1 \\ 0 & Med & 0.1 \leq U \leq 1.0 \\ 1 & Hi & 1.0 \leq U \leq 3.0 \\ 2 & Vhi & U > 3.0 \end{cases} \quad l = \begin{cases} 0 & Lo & U < 0.1 \\ 1 & Med & 0.1 \leq U \leq 3.0 \\ 2 & Hi & U > 3.0 \end{cases}$$

$$k = \begin{cases} 0 & Lo & U < 1.0 \\ 1 & Med & 1.0 \leq U \leq 3.0 \\ 2 & Hi & U > 3.0 \end{cases} \quad j = \begin{cases} 0 & Lo & U < 1.0 \\ 1 & Hi & U \geq 1.0 \end{cases}$$

The membership boundaries are fuzzified by using the following functions:

a) Triangular function

$$T(u; a, b, c) = \begin{cases} 0 & \text{for } u < a \\ (u-a)/(b-a) & \text{for } a \leq u \leq b \\ (c-u)/(c-b) & \text{for } b \leq u \leq c \\ 0 & \text{for } u > c \end{cases}$$

b) Trapezoidal function

$$\text{Trapezoid } \Pi(u; a, b, c, d) = \begin{cases} 0 & \text{for } u < a \\ (u-a)/(b-a) & \text{for } a \leq u < b \\ 1 & \text{for } b \leq u \leq c \\ (d-u)/(d-c) & \text{for } c < u \leq d \\ 0 & \text{for } u > d \end{cases}$$

c) Linear function declining (L-function)

$$L(u; a, b) = \begin{cases} 1 & \text{for } u < a \\ (u-a)/(b-a) & \text{for } a \leq u \leq b \\ 0 & \text{for } u > b \end{cases}$$

d) Linear function increasing ( $\Gamma$ -function)

$$\Gamma(u; a, b) = \begin{cases} 0 & \text{for } u < a \\ (u-a)/(b-a) & \text{for } a \leq u \leq b \\ 1 & \text{for } u > b \end{cases}$$

An example of fuzzy membership function for the Roger's 4 ratio input classifications is illustrated in Figure 2.

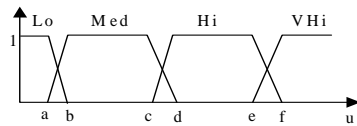


Fig 2: Input code i membership function for Roger's Ratio Method

Fuzzy inference consists of two components which are the antecedent (IF part) and the consequent (THEN part). Here, the fuzzy inference rules are based on the fault interpretations given in Table 3. 18 inference rules can be derived out of the total of 72 possible rules (4x3x3x2). With the fuzzy logic technique, the partial membership may improve the number of matched cases as compared to the ordinary crisp set theory. The following are some examples of the fuzzy rules:

Rules 1: IF  $i=Med$  AND  $j=Lo$  AND  $k=Lo$  AND  $l=Lo$  THEN  $Faults(1)$

Rules 3: IF  $i=Hi$  AND  $j=Lo$  AND  $k=Lo$  AND  $l=Lo$  THEN  $Faults(3)$  Rules 4: IF  $i=VHi$  AND  $j=Lo$  AND  $k=Lo$  AND  $l=Lo$  THEN  $Faults(3)$

The output of the fuzzy inference can be obtained using the Mamdani's Max-Min composition technique. Here the logical 'AND' is replaced with the minimization operator and the logical 'OR' is replaced with the maximization operator [3]. Based on the Roger's ratio rules, the following are some examples of the diagnosed equations:

$$Fault(1) = \min[i=Med, j=Lo, k=Lo, l=Lo]$$

$$Fault(3) = \max\{ \min[i=Hi, j=Lo, k=Lo, l=Lo], \min[i=Med, j=Lo, k=Lo, l=Lo] \}$$

ii. IEC Ratio Fuzzy System

This system has 3 ratios as inputs and 10 conditions as outputs (including "no prediction" output and the 9 conditions from Table 4). The 3 ratios are simplified and classified as either Low (Lo), Medium (Med) or High (Hi) according to membership intervals as defined below:

$$l = C_2H_2 / C_2H_4 = \{Lo, Med, Hi\}$$

$$i = CH_4 / H_2 = \{Lo, Med, Hi\}$$

$$k = C_2H_4 / C_2H_6 = \{Lo, Med, Hi\}$$

$$i = \begin{cases} 0 & Lo & U < 0.1 \\ 1 & Med & 0.1 \leq U \leq 3.0 \\ 2 & Hi & U > 3.0 \end{cases} \quad i = \begin{cases} 1 & Lo & U < 0.1 \\ 0 & Med & 0.1 \leq U \leq 1.0 \\ 2 & Hi & U > 1.0 \end{cases}$$

$$k = \begin{cases} 0 & Lo & U < 1.0 \\ 1 & Med & 1.0 \leq U \leq 3.0 \\ 2 & Hi & U > 3.0 \end{cases}$$

TABLE 4  
CLASSIFICATION OF FAULTS BASED ON IEC RATIO CODES [5]

l	i	k	Characteristic fault
0	0	0	Normal ageing
*	1	0	Partial discharge of low energy density
1	1	0	Partial discharge of high energy density
1-2	0	1-2	Discharge of low energy (Continuous sparking)
1	0	2	Discharge of high energy (Arc with power flow through)
0	0	1	Thermal fault <150 <sup>o</sup> C
0	2	0	Thermal fault 150 <sup>o</sup> -300 <sup>o</sup> C
0	2	1	Thermal fault 300 <sup>o</sup> C-700 <sup>o</sup> C
0	2	2	Thermal fault >700 <sup>o</sup> C

The types of fuzzy membership functions used are the same as the previous method. Only 11 inference rules out of 27 possible rules (3x3x3) can be derived. The inference and diagnosis vector for this method was developed using the same technique as for the Roger's Ratio method.

iii. Doernenburg Ratio Fuzzy System

This method first checks the measured concentrations of the key gases against the limits L1 as specified in Table 5(A). If all are within the limits then the diagnosis would be "normal". Otherwise, it indicates a fault condition, and 4 ratios are then calculated based on the gas concentrations and used to classify the fault.

TABLE 5:  
CONCENTRATION L1 (A) AND FAULT DIAGNOSIS FOR  
DOERNENBURG RATIO METHOD (B) [8]

Key Gas	Concentrations L1 (ppm)
Hydrogen (H <sub>2</sub> )	100
Methane (CH <sub>4</sub> )	120
Carbon Monoxide (CO)	350
Acetylene (C <sub>2</sub> H <sub>2</sub> )	35
Ethylene (C <sub>2</sub> H <sub>4</sub> )	50
Ethane (C <sub>2</sub> H <sub>6</sub> )	65

(A)

Suggested Fault Diagnosis	Ratio 1 (R1) CH <sub>4</sub> /H <sub>2</sub>		Ratio 2 (R2) C <sub>2</sub> H <sub>2</sub> /C <sub>2</sub> H <sub>4</sub>		Ratio 3 (R3) C <sub>2</sub> H <sub>2</sub> /CH <sub>4</sub>		Ratio 4 (R4) C <sub>2</sub> H <sub>6</sub> /C <sub>2</sub> H <sub>2</sub>	
	Extracted From Oil Gas Space	Extracted From Oil Gas Space	Extracted From Oil Gas Space	Extracted From Oil Gas Space	Extracted From Oil Gas Space	Extracted From Oil Gas Space	Extracted From Oil Gas Space	
1-Thermal Decomposition	>1.0	>0.1	<0.75	<1.0	<0.3	<0.1	>0.4	>0.2
2-Corona (Low Intensity PD)	<0.1	<0.01	Not Significant		<0.3	<0.1	>0.4	>0.2
3-Arcing (High Intensity PD)	>0.1	>0.01	>0.75	>1.0	>0.3	>0.1	<0.4	<0.2

(B)

The 4 ratios give 8 different input parameters as referred to Table 5(B) (4 for first column and 4 for second column) and 5 conditions (normal, thermal, corona, no prediction and arcing) as output parameter. The input ratios are classified as either Low, Medium or High according to membership intervals as defined below:

$$\begin{aligned}
 R1 &= CH_4 / H_2 \\
 R2 &= C_2H_2 / C_2H_4 \\
 R3 &= C_2H_6 / CH_4 \\
 R4 &= C_2H_6 / C_2H_2
 \end{aligned}$$

$$\begin{aligned}
 R11 &= \{Lo, Med, Hi\} & R12 &= \{Lo, Med, Hi\} \\
 R21 &= \{Lo, Hi\} & R22 &= \{Lo, Hi\} \\
 R31 &= \{Lo, Hi\} & R32 &= \{Lo, Hi\} \\
 R41 &= \{Lo, Hi\} & R42 &= \{Lo, Hi\}
 \end{aligned}$$

$$\begin{aligned}
 R11 &= \begin{cases} 0 & Lo & U < 0.1 \\ 1 & Med & 0.1 \leq U \leq 1.0 \\ 2 & Hi & U > 1.0 \end{cases} & R12 &= \begin{cases} 0 & Lo & U < 0.01 \\ 1 & Med & 0.01 \leq U \leq 0.1 \\ 2 & Hi & U > 0.1 \end{cases} \\
 R21 &= \begin{cases} 0 & Lo & U < 0.75 \\ 1 & Hi & U > 0.75 \end{cases} & R22 &= \begin{cases} 0 & Lo & U < 1.0 \\ 1 & Hi & U > 1.0 \end{cases} \\
 R31 &= \begin{cases} 0 & Lo & U < 0.3 \\ 1 & Hi & U > 0.3 \end{cases} & R32 &= \begin{cases} 0 & Lo & U < 0.1 \\ 1 & Hi & U > 0.1 \end{cases} \\
 R41 &= \begin{cases} 0 & Lo & U < 0.4 \\ 1 & Hi & U > 0.4 \end{cases} & R42 &= \begin{cases} 0 & Lo & U < 0.2 \\ 1 & Hi & U > 0.2 \end{cases}
 \end{aligned}$$

The fuzzifying membership functions are the same types used in the Roger's Ratio method. The fuzzy inference rules are based on the fault interpretation shown in Table 5(B). In this case, only 6 fuzzy inferences (3 for first column and 3 for second column) can be derived out of the total of 48 possible rules (3x2x2x2 + 3x2x2x2).

iv. Duval Triangle Ratio Fuzzy System

This system consists of 3 gas percentages as the inputs and the 7 regions in the Duval triangle as the outputs. Similar to the approach used in the previous method, at least one of the gas values must exceed a specified limit (L1) in order to be considered as having a fault. Table 6 lists the gas limiting values for the Duval Triangle method.

TABLE 6:  
L1 LIMITS FOR DUVAL TRIANGLE METHOD[9]

Gas	L1 Limits
H <sub>2</sub>	100
CH <sub>4</sub>	75
C <sub>2</sub> H <sub>2</sub>	3
C <sub>2</sub> H <sub>4</sub>	75
C <sub>2</sub> H <sub>6</sub>	75
CO	700
CO <sub>2</sub>	7,000

The 3 gas percentages are divided into intervals: Z0 to Z7 for CH<sub>4</sub> percentage, S0 to S6 for C<sub>2</sub>H<sub>4</sub> percentage, and P0 to P7 for C<sub>2</sub>H<sub>2</sub> percentage. These are shown in Figure 3 and defined as follows:

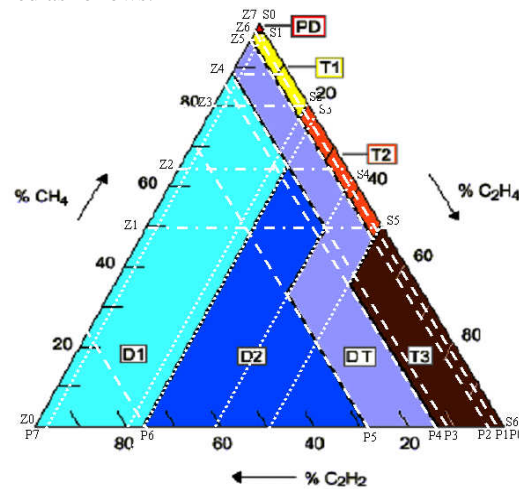


Fig 3: Duval Triangle classification.

$$\begin{aligned}
 \% CH_4 &= \begin{cases} Z1 & U < 50 \\ Z2 & 50 \leq U < 63 \\ Z3 & 63 \leq U < 80 \\ Z4 & 80 \leq U < 88 \\ Z5 & 88 \leq U < 96 \\ Z6 & 96 \leq U < 98 \\ Z7 & U \geq 98 \end{cases} & \% C_2H_4 &= \begin{cases} S1 & U < 2 \\ S2 & 2 \leq U < 20 \\ S3 & 20 \leq U < 23 \\ S4 & 23 \leq U < 37 \\ S5 & 37 \leq U < 50 \\ S6 & U \geq 50 \end{cases} \\
 \% C_2H_2 &= \begin{cases} P1 & U < 2 \\ P2 & 2 \leq U < 4 \\ P3 & 4 \leq U < 12 \\ P4 & 12 \leq U < 14 \\ P5 & 14 \leq U < 28 \\ P6 & 28 \leq U < 77 \\ P7 & U \geq 77 \end{cases}
 \end{aligned}$$

The types of fuzzy membership function used for this method are the same as for previous methods. Based on membership functions of the inputs, this system can have up to 294 rules (7x6x7). However as the total % CH<sub>4</sub> + % C<sub>2</sub>H<sub>4</sub> + % C<sub>2</sub>H<sub>2</sub> must be 100, some rules are not used. The rules were developed based on the regions shown in Figure 4.

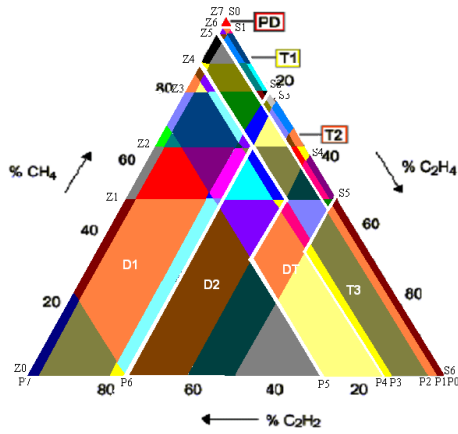


Figure 4: Regions used to develop Duval Triangle Fuzzy system

Only 70 rules were created for this method. These are based on the rules covering each region in Figure 4 as listed below:

- D1 = 24 rules
- D2 = 7 rules
- DT=13 rules
- T1 = 10 rules
- T2 = 11 rules
- T3 = 4 rules
- PD = 1 rules
- Total = 70 rules

The rule components and the output inferences of this method were derived using the same technique as the previous method.

v. Key Gas Fuzzy System

This system is based on the values of the fault gases when at least one exceeds the threshold value. Here all five key gases were used as inputs and the output is the 5 fault types as classified in Table 2. The membership of the fuzzy set “Lo” “Med” or “Hi” was used for each fault gas.

vi. Logarithmic Nomograph Fuzzy System

This method combines the fault gas ratio concept with the Key Gas threshold value. It consists of a series of vertical logarithmic scales representing the concentrations of the individual gases. Straight lines are drawn between adjacent scales to connect the points representing the values of the individual gas concentration. The diagnostic criteria for determining the type of fault are based on the slopes of these lines. There are 28 fuzzy inference rules (2x2x7 which is type of slope multiplied with type of faults and number of vertical axes). The membership functions are the linear type L-function and Γ -function.

III. RESULTS

Each method was tested against all the 69 cases in the data set. The percentages of successful prediction and consistency are calculated using the following formulas:

$$S_{Fn} = \frac{R_{Fn}}{\text{Number of cases of } Fn} \times 100 \quad (1)$$

$$C_{Fn} = \frac{\sum_{n=1}^{n=5} S_{Fn}}{\text{Number of fault types}} \times 100 \quad (2)$$

where:

$F_n$  = fault type code (n=1,2,3,4,5)

TABLE 7: RESULT ANALYSIS FOR EACH TYPE OF FAULTS WITHOUT FUZZY (WF) AND WITH FUZZY SYSTEM (FS).

Method	Faults Code	Number of predictions (P)		Number of correct predictions (R)		% Successful prediction (S)		Consistency (C)	
		WF	FS	WF	FS	WF	FS	WF	FS
Roger	F <sub>1</sub>	1	1	0	0	0%	0%	26%	30%
	F <sub>2</sub>	13	16	13	16	39%	48%		
	F <sub>3</sub>	13	17	12	14	55%	64%		
	F <sub>4</sub>	3	3	3	3	38%	38%		
	F <sub>5</sub>	1	1	0	0	0%	0%		
IEC	F <sub>1</sub>	1	1	0	1	0%	0%	29%	40%
	F <sub>2</sub>	14	26	14	26	42%	79%		
	F <sub>3</sub>	17	19	15	18	68%	82%		
	F <sub>4</sub>	1	3	1	3	13%	38%		
	F <sub>5</sub>	1	1	1	0	20%	0%		
Nomograph	F <sub>1</sub>	6	6	0	0	0%	0%	68%	68%
	F <sub>2</sub>	23	23	21	21	64%	64%		
	F <sub>3</sub>	19	19	17	17	77%	77%		
	F <sub>4</sub>	15	15	8	8	100%	100%		
	F <sub>5</sub>	6	6	5	5	100%	100%		
Doernenburg	F <sub>1</sub>	0	0	0	0	0%	0%	36%	63%
	F <sub>2</sub>	15	23	15	21	45%	64%		
	F <sub>3</sub>	9	13	8	13	36%	59%		
	F <sub>4</sub>	1	1	0	1	0%	13%		
	F <sub>5</sub>	6	15	5	5	100%	100%		
Duval	F <sub>1</sub>	2	2	1	1	100%	100%	82%	84%
	F <sub>2</sub>	31	31	29	28	88%	85%		
	F <sub>3</sub>	26	26	21	22	95%	100%		
	F <sub>4</sub>	4	4	2	3	25%	38%		
	F <sub>5</sub>	6	6	5	5	100%	100%		
Key Gas	F <sub>1</sub>	2	2	1	1	100%	100%	77%	77%
	F <sub>2</sub>	48	48	33	33	100%	100%		
	F <sub>3</sub>	11	11	10	10	45%	45%		
	F <sub>4</sub>	3	3	3	3	38%	38%		
	F <sub>5</sub>	5	5	5	5	100%	100%		

The results are presented in Table 7. For systems without fuzzy logic, it clearly shows that those methods which take into account the limit value of fault gases before doing diagnosis have better success in predicting the normal condition. On the other hand, methods that have no limit values of fault gases always fail to predict the normal condition. This affects the consistency result. Note the low consistency value (<50%) with some of the methods. It is found that the Duval Triangle method is the most consistent method followed by the Key Gas, Nomograph, Doernenburg, IEC Ratio and lastly the Roger Ratio method (when tested without using fuzzy logic).

By applying fuzzy logic, the analysis shows that the consistencies of most methods are improved. The exceptions are the Nomograph and the Key Gas method where consistencies remain at the same values. The Duval Triangle method is still the most consistent method with 2% consistency improvement followed by Key Gas with no improvement, Nomograph with no improvement, Doernenburg with 27% improvement, IEC with 11% improvement and lastly Roger’s method with 4% improvement.

It was also found that for both types of systems, the best methods for predicting fault types F1 and F2 are the Duval Triangle and the Key Gas method. The Duval Triangle

method also is the best method for predicting fault types F3 and F5. Other than the Duval Triangle, the Nomograph is the best method for predicting fault types F5 and F4.

In addition to consistency, the accuracy of each method is another parameter used for comparison. Here, the accuracy is calculated in two different ways:  $A_p$  when considering only the predicted cases  $T_p$ , and  $A_T$  when considering the total number of cases  $T_C$ . Their formulas are:

$$A_p = \frac{T_R}{T_P} \times 100 \quad (3)$$

$$A_T = \frac{T_R}{T_C} \times 100 \quad (4)$$

TABLE 8:  
COMPARISON OF ACCURACY VALUES.

	Roger		IEC		Nomograph		Doernenburg		Duval		Key Gas	
	WF	FS	WF	FS	WF	FS	WF	FS	WF	FS	WF	FS
Total cases, $T_C$	69	69	69	69	69	69	69	69	69	69	69	69
No predictions, $T_{NP}$	38	31	35	19	0	0	38	15	0	0	0	0
Number of predictions, $T_P$	31	38	34	50	69	69	31	54	69	69	69	69
Correct predictions, $T_R$	28	33	31	47	51	51	28	40	58	57	52	52
Incorrect predictions, $T_W$	3	5	3	3	18	18	3	14	11	10	17	17
% Accuracy (predicted cases), $A_p$	90	87	91	94	74	74	90	74	84	86	75	75
% Accuracy (total cases), $A_T$	41	48	45	68	74	74	41	58	84	86	75	75

The results are summarized in Table 8. Without using fuzzy logic, the results based on the predicted cases show that all methods have accuracy more than 70 percent. The most accurate is the IEC Ratio method followed by the Roger Ratio, Doernenburg, Duval Triangle, Key Gas and Nomograph method. As can be seen from the table, those methods that used specific code in their diagnosis have high accuracy (>90%). On the other hand, methods that use direct interpretation based on each value of fault gases are less accurate. However, the accuracy based on the total number of cases shows a different trend. Because of the high number of cases with no prediction, the accuracy drops significantly (<70%) for methods that used specific codes in the diagnosis.

When fuzzy logic was applied, the accuracy based on the total number of cases improves as the number of 'no predictions' is now smaller. The accuracy of the Roger Ratio method has increased by 7%, IEC method by 23%, Doernenburg by 17% and Duval Triangle by 2%. As Nomograph and Key Gas are the two methods that use direct values of fault gases, the application of fuzzy logic does not affect their accuracy values.

#### IV. CONCLUSION

In this paper, a comprehensive investigation of various methods for interpreting DGA results was carried out and the possibility of improving the diagnosis with the aid of

fuzzy logic was explored. The results show that those methods using specific codes in their interpretation have higher value of accuracy for predicted cases in both systems (with or without fuzzy logic) when compared to others. However, the accuracy is somewhat worsened when fuzzy system was applied. This is because the number of predictions when using fuzzy system is increased and this increases the possibility of incorrect predictions. However, there are some improvements on the accuracy based on total cases and the consistency when using fuzzy system for these methods even though the values are still less than other methods.

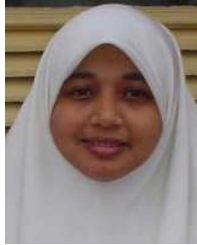
The reversed results were found for those methods that use direct values of fault gases in their interpretation as against methods that use specific codes. These methods have higher consistency and accuracy based on total cases but have low values of accuracy based on predicted cases as compare to other methods in both systems. This is expected because they attempt to provide predictions for all cases. But as they have all the interpretations, the prediction is likely to be incorrect for certain cases. Indeed for these methods, the application of fuzzy system does not improve the diagnosis results. This is because these methods are direct methods and do not have multiple numeric thresholds and gas ratio boundaries that can be improved by applying fuzzy membership function.

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#### VI. BIOGRAPHIES



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# Risk-constrained Optimal Bidding Strategies for Generation Companies using Differential Evolution

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**Abstract**— The emerging electricity market behaves more like an oligopoly than a perfectly competitive market. The profit of each supplier is influenced to varying extents by differences in the degree of imperfection of knowledge of their rivals. This paper discusses the optimal bidding strategies of Generating companies (Gencos) solved using Differential Evolution (DE) algorithm for the first time. It is assumed that each Genco bids a linear supply function, and chooses the coefficient in the linear supply function to maximize their benefits, subject to expectations about how the rivals will bid. A normal probability distribution function (pdf) is used to describe the bidding behaviors of rivals and the problem of building optimal bidding strategies for Gencos is formulated as stochastic optimization problem. The proposed algorithm is tested for six generating companies with different risk coefficients and load price elasticity factor. The obtained results are compared with those obtained by Reference [14].

**Keywords** – Electricity market, Gencos (Generating Companies), Bidding Strategy, Stochastic Optimization, Monte Carlo Method, Differential Evolution.

## I. INTRODUCTION

In the past several years, the power industry, in many countries around the globe, has been undergoing massive changes to introduce competition. Accordingly, a variety of restructuring models have been proposed, considered and experimented with in different countries. Among these models, the power pool (Pool co-type) market structure is the most popular.

The power pool acts, effectively, like a broker for managing energy suppliers, bidders and large customers, and establishes a market clearing price (MCP). MCP is the bid price of the most expensive supplier that is needed to completely meet the demand and is used as the basis for the settlement of market commitments. Regardless of the bidding prices from suppliers, all selected bidders are paid the MCP. This approach is adopted to encourage suppliers in a competitive market to price energy close to their marginal costs. The sealed bid auction is widely used in the pool-co type electricity market. Each supplier submits a sealed bid to the pool to compete for the supply of the forecast load that is broadcast by the pool. Theoretically, in a perfectly competitive market, suppliers should bid at, or very close to their marginal production costs to maximize returns. However, the electricity market is not perfectly competitive

due to special features, such as large investment size (barrier to entry) and economy of scale in the generation sector, and therefore more akin to oligopoly.

In oligopolistic electricity markets, Gencos could exercise strategic bidding to maximize their own profits. The problem of how to develop optimal bidding strategies for competitive Gencos in the electricity market environment was addressed for the first time in [1]. A comprehensive review of optimal bidding strategies in Electricity market has been reported in [2]. The main factors which affect the bidding behavior are the demand variation, generator production cost, operating (or) some regulatory constraints and other competitors bidding behavior etc. Among them, the most uncertain factor is rivals bidding behavior that compounds the difficulties in bidding strategy decision process due to special nature of electricity compared to the other commodities where each player tries to play game to maximize their own profit.

At present most of the research work is based on the estimation of rival's bidding behaviors by employing available information such as historical bidding data. Bidding decision based on incomplete information will surely incur certain risks to the Gencos concerned. For example, if the estimations of a Genco about rival are higher than their actual bidding prices and this will lead to the risk of not being dispatched or the dispatched generation level significantly lower than expected.

David and Wen [3]-[5], have modeled the strategic bidding as a stochastic optimization problem for single period auction where competitor's bidding behavior is estimated through stochastic Monte Carlo simulation. Monte Carlo simulation [6] is a technique which provides probabilistically approximate solutions to mathematical, physical and engineering problems by performing stochastic simulation using random numbers. It can be directly applied

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to problems with inherent probabilistic structures and requires the physical or mathematical system to be described by probability density functions.

Markov decision process was applied in [7] to solve multi-stage probabilistic bidding decision problem. Richter and Sheble have applied genetic algorithm GA [8]-[9] to Genco strategies and schedules in which an intelligent bidding strategy was developed using GP-Automata algorithm for the bidding strategy. Although GA has an advantage of searching the solution space more thoroughly, they are sensitive to the choice of parameter such as the cross-over parameter and mutation probabilities. The other frequently applied AI techniques [10]-[12] such as Simulated Annealing (SA), Evolutionary Programming (EP) and Particle Swarm Optimization (PSO) also suffer from proper selection of parameter such as temperature in SA, scaling factor in EP, and inertia weight and learning factors in PSO. Bajpai et al., [13] have studied the bidding strategy in uniform price spot market using Fuzzy Adaptive Particle swarm Optimization. Ma et al., [14] have developed the risk constrained optimal bidding strategies for single sided auction using an optimization based method.

Although the importance of risk management in bidding decision-making is widely recognized [14], very limited research work has been done in this field up to now and the research outputs achieved so far are very preliminary.

This paper suggests an alternative framework of developing optimal bidding strategies for Gencos in a deregulated market with associated risks taken into account. A normal probability distribution function (pdf) is used to describe the bidding behaviors of rivals. The problem is stochastic in nature which is solved by Monte-Carlo method and the optimal solution is obtained by Differential Evolution method [16] for the first time. Differential evolution method, a modified GA, which is an efficient and robust method, is used that can generate better optimal solution in less calculation time with stable convergence characteristics compared to other population based methods. The technique presented in this paper can be adapted to the more complex situation, and this will be accounted for in later studies.

This paper is organized as follows. Section II explains the formulation of the problem for the single sided auction. Section III describes the proposed solution algorithm using Monte Carlo and Differential evolution methods. Section IV illustrates conceptual analysis for a test system. The results and discussions are presented in the Section V. Conclusive remarks are given in section VI.

## II. PROBLEM FORMULATION

Consider there are  $n$  independent Gencos participating in a pool based single-buyer electricity market in which the sealed auction with a uniform MCP is employed. Assume that each Genco is required to submit a linear supply function to the pool together with the generation output limits and the  $j^{th}$  Genco's supply function is  $B_j(P_j) = \alpha_j + \beta_j P_j$ , where  $P_j$  is the generation output and  $\alpha_j$  and  $\beta_j$  are the bidding coefficients. In a well competitive situation, the bidding coefficients will be equal to the production cost coefficients. Generation output limits

$P_{j_{max}}$  and  $P_{j_{min}}$  are also specified by the bidder. Upon receiving bids from Gencos, the pool determines a set of generation outputs that meets the load demand and minimizes the total purchasing cost. It is clear that dispatch of the generated power should satisfy the following Eqn. (1)-(3).

$$\alpha_j + \beta_j P_j = R \quad (j = 1, 2, \dots, n) \quad (1)$$

$$\sum_{j=1}^n P_j = Q(R) \quad (2)$$

$$P_{j_{min}} \leq P_j \leq P_{j_{max}} \quad (3)$$

where,  $R$  is the market clearing price, and  $Q(R)$  is the total demand given by

$$Q(R) = Q_0 - KR \quad (4)$$

$K$  is a nonnegative constant used to represent the load – price elasticity. When the inequality constraints (3) are ignored, the solutions to (1) and (2) are,

$$R = \frac{Q_0 + \sum_{j=1}^n (\alpha_j / \beta_j)}{K + \sum_{j=1}^n (1 / \beta_j)} \quad (5)$$

$$P_j = \frac{(R - \alpha_j)}{\beta_j} \quad (6)$$

The calculated values of  $P_j$  should be checked for violation of limits as follows

$$\begin{aligned} &\text{if } P_j < P_{j_{min}} \\ &\text{set } P_j = 0 \end{aligned}$$

and the Genco  $j$  should be removed from the competition since the dispatch is less than the minimum output of the supplier and

$$\begin{aligned} &\text{if } P_j > P_{j_{max}} \\ &\text{set } P_j = P_{j_{max}} \end{aligned}$$

and fix the dispatched level of this generator since it is no longer a marginal unit. Repeat this procedure until the dispatched levels of all generators are fixed and no constraints are violated. In this case we have not considered the transmission losses into account.

The profit of Genco  $i$  ( $i = 1, 2, \dots, n$ ) in a unit time can be described as

$$\pi_i = RP_i - C_i(P_i) \quad (7)$$

If  $P_i$  does not violate the constraints (3), the eqn. (7) can be rewritten as

$$\pi_i = \frac{R(R - \alpha_i)}{\beta_i} - C_i \left( \frac{R - \alpha_i}{\beta_i} \right) \quad (8)$$

where  $C_i(P_i)$  is the production cost function of the  $i^{th}$  Genco.

Since the  $i^{th}$  Genco does not know the rivals bidding price before the auction, the optimization problem of maximizing the profit of Genco is not possible. But the rival's bidding behavior can however be estimated using the historical data, load forecast and any other available information.

We assumed that the rival's bidding coefficients  $\alpha_j$  and  $\beta_j$  ( $j \neq i$ ) as estimated by  $i^{th}$  Genco, obey a jointly normal distribution as follows,

$$(\alpha_j, \beta_j) \square N \left\{ \begin{bmatrix} \mu_j^{(\alpha)} \\ \mu_j^{(\beta)} \end{bmatrix}, \begin{bmatrix} (\sigma_j^{(\alpha)})^2 & \rho_j \sigma_j^{(\alpha)} \sigma_j^{(\beta)} \\ \rho_j \sigma_j^{(\alpha)} \sigma_j^{(\beta)} & (\sigma_j^{(\beta)})^2 \end{bmatrix} \right\} \quad (9)$$

where,  $\rho_j$  is the correlation coefficient between  $\alpha_j$  and  $\beta_j$ . The marginal distributions of  $\alpha_j$  and  $\beta_j$  are both normal with mean values  $\mu_j^{(\alpha)}$ ,  $\mu_j^{(\beta)}$ , and  $\sigma_j^{(\alpha)}$ ,  $\sigma_j^{(\beta)}$ , respectively.

From the well developed investment theory, it is known that the variance of the potential profit could be used to evaluate the risk of an investment. Following this concept, the problem of building an optimal bidding strategy for the  $i^{th}$  Genco with associated risks taken into account could be formulated as the following stochastic optimization problem. Maximize

$$\Psi(\alpha_i, \beta_i) = (1 - \lambda)E(\pi_i) - \lambda D(\pi_i) \quad (10)$$

Subject to

$$P_{i,\min} \leq \frac{E(R) - \alpha_i}{\beta_i} \leq P_{i,\max} \quad (11)$$

where  $E(\pi_i)$  and  $D(\pi_i) = \sqrt{\{\text{var}(\pi_i)\}}$  are the expected value and standard deviation of the profit  $\pi_i$  and  $E(R)$  is the expected value of MCP. Risk coefficient  $\lambda$  ( $0 \leq \lambda \leq 1$ ) is used to represent the degree of risk averseness of Genco  $i$ . The value  $\lambda = 0$  denotes the situation that the only objective is to maximize profit without consideration of risks, and  $\lambda = 1$  represents the other extreme where risk minimization is the unique objective. Henceforth Genco should balance these conflicting objectives (i.e.,), profit maximization and risk minimization. Hence the problem of building an optimal bidding strategy for the  $i^{th}$  Genco with risk management can be described as: for a given risk coefficient  $\lambda$ , determine bidding coefficients  $\alpha_i$  and  $\beta_i$  so as to maximize  $\Psi(\alpha_i, \beta_i)$  subject to (11).

While maximizing  $\Psi(\alpha_i, \beta_i)$ , with the constraint (11), since two bidding coefficients cannot be optimized at the same time, one of the coefficients is fixed and the other can be searched using any optimization procedure. In this work,

we fixed  $\alpha_i$  and the values of  $\beta_i$  are searched through Differential Evolution technique [16].

### III. PROPOSED ALGORITHM

The stochastic optimization problem formulated in the previous section has been solved using Monte-Carlo simulation, a technique which obtains a probabilistic approximation of a mathematical problem by using statistical sampling technique. It performs stochastic simulation using random numbers and repeatedly calculates the equations to arrive at a solution. A recent population based heuristic algorithm, Differential Evolution has been used for the first time in this paper to obtain the optimal bidding strategy of a Generating Company.

#### A. A Brief Introduction to the Monte Carlo method

The Monte Carlo method provides approximate solutions to a variety of mathematical problems by performing statistical sampling experiments on a computer. This method can be directly applied to problems with inherent probabilistic structures. It requires that the physical or mathematical system be described by probability density functions (pdf's). Simulation can be done by random sampling from these pdf's and necessitates a fast and effective way to generate random numbers uniformly distributed in the interval [0, 1]. The outcomes of these trials are accumulated or tallied. Many simulations are performed and the desired result is taken as an average or expectation over all the trials.

#### B. Differential Evolution

Differential Evolution (DE) is an optimization algorithm developed by Storn and Price, which solves real-valued problems based on the principles of natural evolution [16]. DE uses a population  $P$  of size  $N_p$ , composed of floating point encoded individuals that evolve over  $G$  generations to reach an optimal solution. Each individual  $X_i$  is a vector that contains as many parameters as the problem decision variables  $D$ . The population size  $N_p$  is an algorithm control parameter selected by the user which remains constant throughout the optimization process.

$$X_i^{(G)} = [X_{1,i}^{(G)}, \dots, X_{D,i}^{(G)}]^T, \quad i = 1, \dots, N_p \quad (12)$$

The optimization process in differential evolution is carried out with three basic operations viz, mutation, crossover and selection. This algorithm starts by creating an initial population of  $N_p$  vectors. Random values are assigned to each decision parameter in every vector according to

$$X_{j,i}^{(0)} = X_j^{\min} + \eta_j (X_j^{\max} - X_j^{\min}) \quad (13)$$

where  $i = 1, \dots, N_p$  and  $j = 1, \dots, D$ ;  $X_j^{\min}$  and  $X_j^{\max}$  are the lower and upper bounds of the  $j^{th}$  decision parameter; and  $\eta_j$  is an uniformly distributed random number within [0,1] generated a new for each value of  $j$ .  $X_{j,i}^{(0)}$  is the  $j^{th}$  parameter of the  $i^{th}$  individual of the initial population.

The mutation operator creates mutant vectors ( $X_i'$ ) by perturbing a randomly selected vector ( $X_a$ ) with the difference of two other randomly selected vectors ( $X_b$  and  $X_c$ ).

$$X_i^{(G)} = X_a^{(G)} + F(X_b^{(G)} - X_c^{(G)}) \quad i = 1, \dots, N_p \quad (14)$$

where  $X_a, X_b$  and  $X_c$ , are randomly chosen vectors  $\in \{1, \dots, N_p\}$  and  $a \neq b \neq c \neq i$ .  $X_a, X_b$  and  $X_c$  are selected newly for each parent vector. The scaling constant ( $F$ ) is an algorithm control parameter used to control the perturbation size in the mutation operator and improve algorithm convergence.

The crossover operation generates trial vectors ( $X_i^*$ ) by mixing the parameters of the mutant vectors with the target vectors ( $X_i$ ), according to a selected probability distribution,

$$X_{j,i}^{(G)} = \begin{cases} X_{j,i}^{(G)} & \text{if } \eta_j \leq C_R \text{ or } j = q, \\ X_{j,i}^{(G)} & \text{otherwise} \end{cases} \quad (15)$$

where  $i = 1, \dots, N_p$  and  $j = 1, \dots, D$ ;  $q$  is a randomly chosen index  $\in \{1, \dots, N_p\}$  which guarantees that the trial vector gets at least one parameter from the mutant vector;  $\eta_j$  is a uniformly distributed random number within  $[0,1]$  generated newly for each value of  $j$ . Crossover constant  $C_R$  is an algorithm parameter that controls the diversity of the population and aids the algorithm to escape from local optima.  $X_{j,i}^{(G)}, X_{j,i}^{(G)}$  and  $X_{j,i}^{(G)}$  are the  $j^{\text{th}}$  parameter of the  $i^{\text{th}}$  target vector, mutant vector, and trial vector at generation  $G$ , respectively.

Finally, the selection operator determines the population by choosing, among the trial vectors and their predecessors (target vectors), those individuals which present a better fitness or are more optimal according to

$$X_i^{(G+1)} = \begin{cases} X_i^{(G)} & \text{if } f(X_i^{(G)}) \leq f(X_i^{(G)}), \quad i = 1, \dots, N_p, \\ X_i^{(G)} & \text{otherwise} \end{cases} \quad (16)$$

The optimization process is repeated for several generations, allowing individuals to improve their fitness as they explore the solution space in the search for optimal values.

DE has three essential control parameters: scaling factor ( $F$ ), crossover constant ( $C_R$ ) and population size ( $N_p$ ).

The scaling factor is a value in the range  $(0, 2)$  that controls the amount of perturbation in the mutation process. The crossover constant is a value in the range  $(0, 1)$  that controls the diversity of the population. The population size determines the number of individuals in the population and provides the algorithm enough diversity to search the solution space [20]. DE offers several variants or strategies for optimization. These can be denoted by DE/x/y/z, where x refers to the vector used to generate mutant vectors, y the number of difference vectors used in the mutations process and z the crossover scheme used in the crossover operation.

Totally ten different working strategies have been proposed by Price & Storn [16].

The algorithm used in this paper is the seventh strategy of DE (i.e.) DE/rand/1/bin in which 'DE' represents differential evolution, 'rand' is any randomly chosen vector for perturbations, '1' represents the number of difference vectors to be perturbed and 'bin' is the binomial type of crossover used. Price and Storn [16], [20]-[21] have given some simple rules for choosing the parameter of DE for any given application. According to them, the following ranges are the good initial estimates while using the strategy DE/rand/1/bin:  $F = [0.5, 0.6]$ ,  $C_R = [0.75, 0.9]$ , and  $N_p = [3 * D, 8 * D]$ . Values of scaling factor lower than 0.5 may result in premature convergence, while greater than 1 tend to slow down convergence speed. To avoid local optima, cross over constant should be reduced to provide more diversity of parameters. Large populations help to maintain diverse individuals but slow down convergence speed. Therefore in order to avoid premature convergence, either  $F$  or  $N_p$  should be increased or  $C_R$  should be decreased. Larger values of  $F$  result in larger perturbations and better probabilities to escape from local optima, while lower  $C_R$  preserves more diversity in the population, thus avoiding local optima.

The solution algorithm used in the present study for solving the optimal bidding problem for Genco  $i$  is as follows

- 1) Specify  $\alpha_i$  for the  $i^{\text{th}}$  Genco,  $Q_0, K, P_{j \min}, P_{j \max}$  ( $j = 1, 2, \dots, n$ ) and the parameters of pdf's of the rivals bidding strategies as (18).
- 2) Create the Differential evolution algorithm whose population members represent  $\beta_i$  for the  $i^{\text{th}}$  Genco in the interval of  $(0.5\beta_i, 5\beta_i)$ .
- 3) Initialize DE population and the maximum generation number  $T_{gen}$ .
- 4) Set the iteration counter  $t_{gen} = 0$ .
- 5) Execute a Monte-Carlo simulation for every member of the population to calculate the expected profit and standard deviation as follows:
  - a. Specify the maximum number of Monte-Carlo simulation,  $MC$ .
  - b. Set the Monte-Carlo simulation counter  $mc=0$ .
  - c. Generate random samples for  $\alpha_j, \beta_j, (j = 1, 2, \dots, n) (j \neq i)$ , based on their pdf's, as given in (9).
  - d. Determine the market clearing price  $R$ , using (5) and calculate  $\pi$  using (7).
  - e. Increment the Monte-Carlo simulation counter  $mc=mc+1$ .
  - f. If  $mc < MC$ , go to c: else go to g.
  - g. Calculate the expectation value  $E(\pi)$  and the standard deviation  $D(\pi)$  and then calculate  $\psi(\alpha_i, \beta_i)$  using (10).

- 6) Use  $\psi(\alpha_i, \beta_i)$  as the fitness function for the population members.
- 7) Increment  $t_{gen} = t_{gen} + 1$ .
- 8) Perform mutation, crossover and selection operation.
- 9) If  $t_{gen} < T_{gen}$ , go to 5c. Else go to 10.
- 10) Obtain the fittest member of the Differential Evolution as the optimal bidding strategy, and print the results.

IV. NUMERICAL EXAMPLE

Consider six independent Genco's participating in an electricity market, and the production cost function of the  $j^{th}$  ( $j = 1, 2, \dots, n$ ) Genco is given by,

$$C_j(P_j) = b_j P_j + \frac{1}{2} c_j P_j^2 \tag{17}$$

The production cost function coefficients and output limits of all six genco's are given in Appendix I. Suppose that the second Genco is our subject of research, and its estimation of the rivals bidding parameters i.e. the expected values and the standard deviations are specified as follows:

$$\left. \begin{aligned} \mu_j^{(\alpha)} &= 1.2b_j, \mu_j^{(\beta)} = 1.2c_j \\ 3\sigma_j^{(\alpha)} &= 0.15b_j, 3\sigma_j^{(\beta)} = 0.15c_j \\ \rho_j &= -0.1 \\ j &= 1, 3, 4, 5, 6 \end{aligned} \right\} \tag{18}$$

The specifications of these parameters are the same as in [3]. From (18) it is known that the expected values of  $\alpha_j$  and  $\beta_j$  are specified 20% above  $b_j$  and  $c_j$  respectively and the standard deviations of  $\alpha_j$  and  $\beta_j$  are specified to make them fall in the domains

$$\begin{aligned} [1.05b_j, 1.35b_j] &= [\mu_j^{(\alpha)} - 3\sigma_j^{(\alpha)}, \mu_j^{(\alpha)} + 3\sigma_j^{(\alpha)}] \text{ and} \\ [1.05c_j, 1.35c_j] &= [\mu_j^{(\beta)} - 3\sigma_j^{(\beta)}, \mu_j^{(\beta)} + 3\sigma_j^{(\beta)}] \end{aligned}$$

respectively. Here  $\rho_j$  is specified to be negative because when a Genco decides to increase one of its bidding coefficients, it is more likely that in a mature electricity market it will decrease rather than increasing the other one. Certainly, it is also possible that  $\rho_j$  takes a positive value for some Gencos who want to change the bidding patterns significantly.

Now for Genco 2 fix  $\alpha_2 = 1.75$  and determine  $\beta_2$  by solving the optimization problem as described by using Differential Evolution method [16]. Obviously,  $\beta_2$  should not be less than  $0.5c_2$ , otherwise it will be a loss. The search domain for  $\beta_2$  is specified to be  $[m_1c_2, m_2c_2]$ , and  $m_1 = 0.5, m_2 = 5$ , since this range is wide enough. The simulation parameters are listed in Appendix II.

V. RESULTS AND DISCUSSION

A. Case 1

In this case, we compare the results of dispatched power of all six gencos. The figure (1) shows the values of dispatched powers obtained by the proposed method against those given in Reference [14]. It is seen from the graph that the dispatched power of genco 2 for a given risk coefficient  $\lambda = 0.5$  and  $K = 20$  is more than the other method proving the effectiveness of the differential evolution method. More the power dispatched, more the profit is and hence the benefit to the consumers is also more.

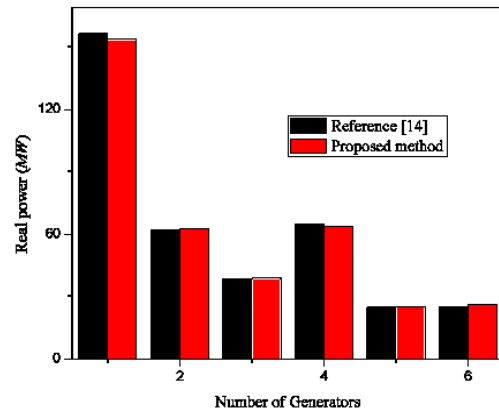


Fig. 1. Optimal Dispatched powers of Generators

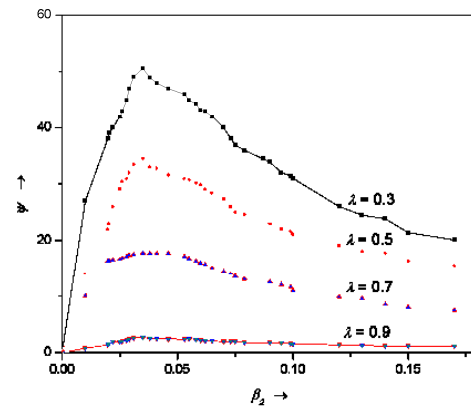


Fig. 2. Variation of profit  $\psi$  with respect to bidding coefficient  $\beta_2$  for different  $\lambda$

B. Case 2

Table I shows the comparative simulation results for proposed method and the optimization based method [14] for various parameters, and it is found that for a given  $\lambda$  DE gives lesser values of the bidding coefficient  $\beta_2$  than the other method, thereby increasing the dispatched power, market clearing price, expected profit and the actual profit. This shows the robustness and effectiveness of the differential evolution method. From Figure 2 it can be clearly seen that when the risk coefficient  $\lambda$  increases the

profit decreases and when it is very high, say  $\lambda = 0.9$  the profit is very less. When  $\lambda$  varies from 0.9 to 0.915 there is a considerable change in the variance of the expected profit which implies that the expected dispatched generation level of a rival is beyond its lower limit and hence the rival quits from the competition. Therefore risk factor should be considered in such a way, that the genco's should survive in the market and also get the maximum benefit compared to their rivals.

### C. Case 3

This case demonstrates the effect of load price elasticity factor, with increase of  $\lambda$ , on different bidding parameters. When the load price elasticity factor  $K$  varies from 15 to 20 and from 20 to 25, with the increase in  $\lambda$ , it can be noted from Tables II-IV, that the expected dispatched power of the genco 2 is increasing, but with a little decrease in the optimal bidding coefficient  $\beta_2$ , expected market clearing price  $R$ , as well as the expected value and variance of the profit.

From Figure 3 it can be noticed that the expected dispatch power level of genco 2 is increasing with increasing  $K$ , whereas for the rivals, the output decreases as  $K$  increases.

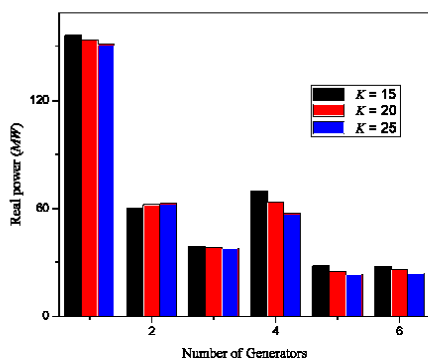


Fig. 3. Dispatched power of six genco's for increased load price elasticity factor

### D. Case 4

In this case the load is varied with 20% increase from the base case and the results are listed in Table V. From the Table V, it can be clearly seen that when the load is 20% more than the base case the expected market clearing price  $R$  is higher than the base case for the constant risk coefficient  $\lambda = 0.5$  and  $K = 20$ . This means that the suppliers have still succeeded in exercising market power. The degree to which the market price of electricity is above the competitive level is 4.14% higher than the base case. But for higher values of  $K$ , when load increases, the bidding coefficient  $\beta_2$ , expected dispatched level  $P_2$ , expected market clearing price  $R$  as well as the expected value and variance of the profit of the genco 2 decrease, thereby facing a more competitive situation to dispatch its power to the consumers.

## VI. CONCLUSIONS

Differential evolution method is proposed, for the first time, to solve the bidding strategies for generation companies

participating in pool-based single buyer electricity markets with risk factor. An example with six suppliers has been used to demonstrate the method, and it has been revealed that the power suppliers can increase their profits by bidding strategically. The results obtained, using Differential evolution, are more optimal compared to those obtained in Reference [14]. The market power of the suppliers is analyzed for different load price elasticity factors. To show the effectiveness of the proposed method, the bidding strategies of the suppliers is also discussed under the increased load condition. The simulation studies were carried out in Matlab environment and executed in Pentium IV processor.

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0	8	3	1			4
0.50	0.0388	60.28	4.085	77.556	28.1490	36.125
0	1	8	0			6
0.70	0.0378	61.59	4.083	77.429	30.6800	19.380
0	8	6	0			0
0.90	0.0366	63.94	4.073	77.113	26.4800	3.0799
0	8	9	0			
0.91	0.0232	94.99	4.012	57.052	19.1650	0.8437
5	9	1	0			

TABLE III: BIDDING STRATEGIES AND ASSOCIATED VARIABLES FOR LOAD PRICE ELASTICITY FACTOR  $K = 20$

$\lambda$	$\beta_2$	$P_2$	$R$	$E(\pi_2)$	$Var(\pi_2)$	$\psi$
0.00	0.0373	61.800	4.044	74.9700	28.010	74.970
0	2	0	1			0
0.30	0.0372	61.830	4.044	74.9342	27.670	50.854
0	4	0	0			2
0.50	0.0371	62.180	4.042	74.9338	26.444	34.883
0	4	0	3			2
0.70	0.0361	63.429	4.039	74.9329	26.293	18.890
0	0	7	9			4
0.90	0.0354	64.334	4.031	74.4361	25.999	2.8584
0	6	9	5			
0.91	0.0232	95.191	3.968	52.7274	17.515	0.8338
5	7	4	9			

TABLE IV: BIDDING STRATEGIES AND ASSOCIATED VARIABLES FOR LOAD PRICE ELASTICITY FACTOR  $K = 25$

$\lambda$	$\beta_2$	$P_2$	$R$	$E(\pi_2)$	$Var(\pi_2)$	$\psi$
0.00	0.03731	61.98	4.030	72.4456	26.8968	72.445
0		0	0			6
0.30	0.03722	62.04	4.002	72.3373	26.8947	49.080
0		6	9			3
0.50	0.03615	62.39	4.002	72.2507	25.5420	33.733
0		6	7			4
0.70	0.03528	63.56	3.993	71.9531	24.0485	18.153
0		6	0			1
0.90	0.03470	64.28	3.992	71.9530	24.0271	2.7335
0		1	9			
0.91	0.02264	95.94	3.927	47.9198	16.6358	0.3411
5	3	9	8			

TABLE V: OPTIMAL RESULTS OF BIDDING PARAMETERS FOR INCREASE IN LOAD

Load	Parameters	$K = 15$	$K = 20$	$K = 25$
540	$\beta_2$	0.3708	0.3698	0.03636
	$P_2$	67.59	66.68	66.464
	$R$	4.25	4.21	4.17
	$E(\pi_2)$	89.61	86.50	83.41
	$Var(\pi_2)$	30.49	30.46	30.42
	$\psi$	42.9	40.49	38.86

TABLE II: BIDDING STRATEGIES AND ASSOCIATED VARIABLES FOR LOAD PRICE ELASTICITY FACTOR  $K = 15$

$\lambda$	$\beta_2$	$P_2$	$R$	$E(\pi_2)$	$Var(\pi_2)$	$\psi$
0.00	0.0394	59.30	4.085	78.112	33.4420	78.112
0	0	0	5			0
0.30	0.0393	59.30	4.085	78.039	32.5421	52.216

VIII. BIOGRAPHIES



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TABLE I: COMPARATIVE RESULTS OF BIDDING STRATEGIES AND ASSOCIATED VARIABLES FOR  $K = 20$

$\lambda$	$\beta_2$		$P_2$		$R$		$E(\pi_2)$		$Var(\pi_2)$		$\Psi$	
	Proposed method	Reference [14]	Proposed method	Reference [14]	Proposed method	Reference [14]	Proposed method	Reference [14]	Proposed method	Reference [14]	Proposed method	Reference [14]
0.00 0	0.037032	0.037042	61.8000	61.78	4.0444	4.0386	74.9700	74.698	28.010	28.309	74.970	74.698
0.50 0	0.037000	0.037041	62.1800	61.79	4.0423	4.0386	74.9338	74.698	26.444	28.309	34.8832	34.829
0.90 0	0.035463	0.037030	64.3349	61.80	4.0315	4.0386	74.4361	74.698	25.999	28.303	2.8584	2.6817
0.91 5	0.023277	--	95.1914	--	3.9689	3.9594	52.7274	45.198	17.515	--	0.8338	--

APPENDIX I

PRODUCTION COST COEFFICIENTS AND GENERATOR OUTPUT LIMITS OF GENCOS

Gencos No	$b_j$ (\$/hr)	$c_j$ (\$/MWhr)	$P_{jmin}$ (MW)	$P_{jmax}$ (MW)
1	2.0	0.00875	50	160
2	1.75	0.035	20	100
3	1.0	0.0625	30	80
4	3.15	0.00334	30	80
5	3.0	0.015	10	60
6	3.0	0.015	10	60

APPENDIX II

SIMULATION PARAMETERS

$C_R$	$F$	$N_p$	$MC$	$T_{gen}$	$Q_0$ (MW)
0.7	0.55	50	10000	100	450